NON-PLANAR INTERFACE CRACK UNDER GENERAL LOADING
1. GLIDE-TYPE EDGE AND SCREW SINUSOIDAL DISLOCATIONS

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ABSTRACT

This study's objective is to analyze the conditions of propagation of an oscillatory front crack along a non-planar interface, under mixed mode I + II + III loading. The crack model consists of a continuous distribution of three families of non-straight dislocations having the shape of the crack front: families 1 and 2 are edges (on average) and family 3 is screw. The associated Burgers vectors \( \mathbf{b}_j \) \((j=\text{I, II, III})\) are directed along the applied tension and shears \( x_2, x_1 \) and \( x_3 \) directions, respectively. The dislocations are aligned along the \( x_3 \) direction and spread in \( x_2x_3 \) planes in a small oscillating form \( \xi(x_1,x_3) \) at an average elevation \( h(x_i) \). In this part I of the study, the displacement and stress fields of dislocations with \( \mathbf{b}_1 \) and \( \mathbf{b}_{\text{III}} \) are given. Results are displayed, that make easily accessible stress terms with singularities \( 1/x_1 \) and \( \delta(x_i) \) (Dirac delta function), involved in the crack analysis to come (Part II of this work).

Keywords : linear elasticity, interface dislocations, Galerkin vector, three-dimensional biharmonic functions, Fourier forms, linear systems of equations.

RÉSUMÉ

Fissure d'interface non plane sous sollicitation extérieure arbitraire
I. Dislocations vis et coins de type glissile

La présente étude se fixe pour objectif d'analyser les conditions de propagation d'une fissure de front oscillatoire le long d'une interface non plane sous sollicitation en mode mixte I+II+III. Le modèle de fissure adopté, est une

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distribution continue de trois familles de dislocations non rectilignes ayant la forme du front de fissure : les familles 1 et 2 sont des coins (en moyenne) et la famille 3 est vis. Les vecteurs de Burgers infinitésimaux associés \( \tilde{b}_j \) \((j= I, II, III)\) sont suivant les directions \( x_2, x_1 \) and \( x_3 \), correspondant à la tension et aux cisaillements appliqués, respectivement. Les dislocations sont suivant la direction \( x_3 \) et s'étalent dans les plans \( x_2x_3 \) dans la forme \( \xi(x_1, x_3) \) à la hauteur \( h(x_i) \). Dans cette partie I de l'étude, les champs de contrainte et de déplacement des dislocations de vecteurs de Burgers \( \tilde{b}_j \) et \( \tilde{b}_{II} \) sont donnés. Les résultats sont présentés de façon à rendre facilement accessibles les termes de contrainte avec les singularités \( 1/x_i \) et \( \delta(x_i) \) (fonction delta de Dirac) impliquées dans l'analyse des fissures (partie II) à venir de notre travail.

**Mots-clés :** élasticité linéaire, dislocations d'interface, Vecteur de Galerkin, fonctions biharmoniques à trois dimensions, expansions en séries de Fourier, systèmes d'équations linéaires.

**I - INTRODUCTION**

The main objective of this study is to analyze the conditions for the propagation of a crack, along a non-planar interface \( R \) of arbitrary shape, in a pair of two firmly welded different solids \( R1 \) and \( R2 \). This work is fundamental by the fact that most of the materials, used in practice, are composite materials that can deteriorate in service by the propagation of interface cracks. To a large crack under general applied loading, fluctuating about average fracture plane (e.g. \( Ox_1x_3 \) of a Cartesian coordinate system \( x_i \)) normal to the applied tension direction, the following description applies locally : \( x_1 \), average crack propagation direction in that plane; \( x_2x_3 \), local plane of the crack front and \( x_3 \), average crack-front direction. Hence, we can define a simple model by specifying that the crack extends from \( x_i = -a \) to \( a \), with a front lying in the \( x_2x_3 \) -plane in a general form \( x_2 = f(x_1, x_3) \) and an average direction that runs indefinitely along \( x_3 \). This is the model of interface crack (Figure 1) that we shall adopt in the present study where \( R1 \) and \( R2 \) are confined for illustration purpose in a parallelepiped of finite sizes. Clearly, this is a model that applies to large cracks that have propagated over large distance and interfaces not far from the average fracture plane.

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Our method of analysis consists of representing the crack by a continuous array of infinitesimal dislocations having the same shape as the crack front. $f$ can be expanded in the form of a Fourier series as

$$
 f = \sum_n \left( \xi_n \sin \kappa_n x_3 + \delta_n \cos \kappa_n x_3 \right) + h \equiv \xi + h
$$

where $n$ is a positive integer; $h$, $\xi_n$, $\delta_n$ and $\kappa_n$ are real numbers that depend on position $x_1$ along the crack length. From the stress fields of the dislocations, the crack-tip stresses and crack extension force (per unit length of the crack front) can be evaluated. Such analyses, with variable complexity of the crack front, exist in the case of an infinitely extended isotropic medium ([1 to 7], among others). In the case of an interface crack, mode I loading causes shear stresses corresponding to mode II and vice versa (for example, see [8, 9]). It is mandatory to have the stress fields of three types of dislocation before undertaking an analysis of the conditions of non-planar interface crack motion. In the present part I of this study, the elastic fields of sinusoidal dislocations, edges and screws with Burgers vectors $\vec{b}_I = (0, b, 0)$ and $\vec{b}_{II} = (0, 0, b)$ are described, with special attention to stress terms with singularities $1/x_1$ and Dirac delta function $\delta(x_i)$ that come into play in crack analyses (calculation of

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the crack extension force, for example). In what follows, the methodology for obtaining the dislocation elastic fields and associated calculation results are presented in Section 2 and 3, respectively. Discussion and concluding remarks form Section 4 where, in particular, the passage from the elastic fields of dislocations with a sinusoidal shape to those having the form $f$ is indicated.

The second part II of the work will deal with the elastic fields of edges (climb-type) with $\vec{b}_{n} = (b,0,0)$, crack-tip stresses and crack extension force when the non-planar interface crack is loaded in mixed mode I+II+III.

II - METHODOLOGY

We consider a dislocation lying on a non-planar interface $S$ having the form of a corrugated sheet that separates two firmly welded elastic solids $S_1$ and $S_2$ of infinite sizes (Figure 2). $S$ is defined by the point $P_3(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ and $S_1$ and $S_2$ occupy the regions $x_2 > \xi_n \sin \kappa_n x_3$ and $x_2 < \xi_n \sin \kappa_n x_3$, respectively. The situation is shown in Figure 2 where $S_1$ and $S_2$ are confined for illustration purpose in a parallelepiped of finite sizes. The dislocation is located at the origin, runs indefinitely in the $x_3$–direction and spreads in the $x_2x_3$–plane in the form

$$A_n = \xi_n \sin \kappa_n x_3.$$  (2)

Figure 2: Two elastic mediums (1) and (2) welded along a non-planar sinusoidal surface and containing an interface sinusoidal dislocation at the origin. The dislocation lies in the $Ox_2x_3$–plane and runs indefinitely in the $x_3$–direction

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When its Burgers vector \( \vec{b}_I = (0, b, 0) \) is in the \( x_2 \) – direction, the dislocation is edge on average. Because \( \vec{b}_I \) is in the plane of location of the dislocation, this is a glide-type edge dislocation. With the Burgers vector \( \vec{b}_{II} = (0,0,b) \) in the \( x_3 \) – direction, the dislocation in Figure 2 is screw on average. This is the aim of the present study to provide displacement \( \vec{u}^{(m)} \) and stress \( (\sigma)^{(m)} \) fields of these types of interface dislocation. The solution methodology is that of \([10, 11]\). The elastic fields \( (\vec{u}^{(m)}, (\sigma)^{(m)}) \) are assumed to be the difference between two quantities \( (\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty}) \) and \( (\vec{u}^{(m)W}, (\sigma)^{(m)W}) \):

\[
\begin{align*}
\vec{u}^{(m)} &= \vec{u}^{(m)\infty} - \vec{u}^{(m)W} \\
(\sigma)^{(m)} &= (\sigma)^{(m)\infty} - (\sigma)^{(m)W} 
\end{align*}
\] (3)

The former with \( \infty \) corresponds to the fields of a sinusoidal dislocation (edge or screw) in an infinitely extended homogeneous solid \( (m) \); the latter with \( W \) satisfies the equations of equilibrium and is constructed in such a way that:

(a) \( (\vec{u}^{(m)}, (\sigma)^{(m)}) \) are continuous at the crossing of the interface, implying that

\[
\begin{align*}
\Delta \vec{u}^{\infty}(P_S) &= \vec{u}^{(2)\infty} - \vec{u}^{(1)\infty} = \vec{u}^{(2)W} - \vec{u}^{(1)W} \equiv \Delta \vec{u}^{(W)}(P_S) \\
(\Delta \sigma)^{\infty}(P_S) &= (\sigma)^{(2)\infty} - (\sigma)^{(1)\infty} = (\sigma)^{(2)W} - (\sigma)^{(1)W} \equiv (\Delta \sigma)^{(W)}(P_S) 
\end{align*}
\] (4)

(b) \( (\vec{u}^{(m)}, (\sigma)^{(m)}) \) tends to \( (\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty}) \) when one moves far away from the interface in the \( x_2 \) – direction. This means that

\[
\begin{align*}
\vec{u}^{(m)W} &\to 0 \\
(\sigma)^{(m)W} &\to 0 
\end{align*}
\] (5)

when \( |x_2| \to \infty \). \( (\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty}) \) may be taken from \([5, 6]\); they are given to linear expressions with respect to \( \xi_n \). The associated terms \( (\vec{u}^{(0)\infty}, (\sigma)^{(0)\infty}) \) of zero order correspond to the fields of a straight dislocation and second terms \( (\vec{u}^{A_n(m)\infty}, (\sigma)^{A_n(m)\infty}) \) are proportional to either \( A_n \) or its spatial derivative \( \partial A_n / x_3 \). Hence we have

\[
\begin{align*}
\Delta \vec{u}^{\infty}(P_S) &= \Delta \vec{u}^{(0)\infty} + \Delta \vec{u}^{A_n\infty} \\
(\Delta \sigma)^{\infty}(P_S) &= (\Delta \sigma)^{(0)\infty} + (\Delta \sigma)^{A_n\infty} 
\end{align*}
\] (6)

on the interface point \( P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3) \); Appendix A below gives the

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complete list, component by component, for the sinusoidal screw dislocation; the corresponding values for the glide-type sinusoidal edge dislocation have been given in [10, 11]. \((\vec{u}^{(m)W}, (\sigma)^{(m)W})\) are obtained with the help of Galerkin vectors; these are available for the edge dislocation (see [10, 11]). For the screw dislocation, we arrive at Galerkin vectors \(\vec{V}\) with only one non-zero \(x_3\) component, arranged in the form

\[
V_3(\vec{x}) = \overline{\alpha}_3(k)e^{\vec{k} \cdot \vec{x}} + \overline{\beta}_3(k)x_2e^{\vec{k} \cdot \vec{x}}
\]  

(7)

under the condition \(k^2 = k_1^2 + k_2^2 + k_3^2 = 0\) that ensures the biharmonicity of \(V_3\).

For \(V_3\) to cancel far from the interface, we write

\[
k_2 = k_2^{(m)} \equiv (-1)^{m-1}i\sqrt{k_1^2 + k_3^2}
\]  

(8)

with \(m = 1\) when \(x_2 > \xi_n\sin\kappa_n x_3\) (half-space 1) and \(m = 2\) when \(x_2 < \xi_n\sin\kappa_n x_3\) (half-space 2). We use the notations

\[
\vec{k}^{(m)} \equiv (k_1,k_2^{(m)},k_3), \quad \overline{\alpha}_3^{(m)} = \overline{\alpha}_3(\vec{k}^{(m)}), \quad \overline{\beta}_3^{(m)} = \overline{\beta}_3(\vec{k}^{(m)});
\]

hence for half-space 1 (\(x_2 > \xi_n\sin\kappa_n x_3\), solid (1)

\[
V_3(\vec{x}) \equiv V_3^{(1)}(\vec{x}) = \overline{\alpha}_3^{(1)}e^{\vec{k}^{(1)} \cdot \vec{x}} + \overline{\beta}_3^{(1)}x_2e^{\vec{k}^{(1)} \cdot \vec{x}}
\]

and for half-space 2 (\(x_2 < \xi_n\sin\kappa_n x_3\), solid (2)

\[
V_3(\vec{x}) \equiv V_3^{(2)}(\vec{x}) = \overline{\alpha}_3^{(2)}e^{\vec{k}^{(2)} \cdot \vec{x}} + \overline{\beta}_3^{(2)}x_2e^{\vec{k}^{(2)} \cdot \vec{x}}
\]

The elastic fields corresponding to \(V_3\) (7) may be first calculated; then, more general forms \(\vec{u}^{(m)V}\) and \((\sigma)^{(m)V}\) are constructed from the previous ones by superposition over \(k_1\) and \(k_3\); we may write

\[
\vec{u}^{(m)V} = \vec{u}^{(m)A} + \vec{u}^{(m)B} = \vec{u}^{(m)A+B}
\]

\[
(\sigma)^{(m)V} = (\sigma)^{(m)A} + (\sigma)^{(m)B} = (\sigma)^{(m)A+B}
\]

(9)

where terms with \(A\) and \(B\) refer to \(\overline{\alpha}_3\) and \(\overline{\beta}_3\) in (7) respectively. For \(\vec{u}^{(m)V}\) and \((\sigma)^{(m)V}\) to conform with \(\vec{u}^{(m)\infty}\) and \((\sigma)^{(m)\infty}\), the summation over \(k_1\) is continuous and that over \(k_3\) is discrete. \(k_3\) takes three values: \(-\kappa_n, 0, \kappa_n\). The

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fields corresponding to \( k_3 = 0 \) are denoted \( \tilde{u}^{(0)(m)V} \) and \( (\sigma)^{(0)(m)V} \) and terms associated with \( k_3 = -\kappa_n \) and \( \kappa_n \) are merged to form expressions denoted by \( \tilde{u}^{A,(m)V} \) and \( (\sigma)^{A,(m)V} \); this is made possible by requiring that

\[
\alpha_3^{(m)}(\kappa_n) = -\alpha_3^{(m)}(-\kappa_n), \quad \beta_3^{(m)}(\kappa_n) = -\beta_3^{(m)}(-\kappa_n). \tag{10}
\]

In (20), \( \alpha_3^{(m)}(\kappa_n) \) stands for \( \alpha_3(k_1, k_2^{(m)}, \kappa_n) \). We write

\[
\tilde{u}^{(m)V} = \tilde{u}^{(0)(m)V} + \tilde{u}^{A,(m)V}
\]

\[
(\sigma)^{(m)V} = (\sigma)^{(0)(m)V} + (\sigma)^{A,(m)V}
\]

introducing subsequently the notation \( \tilde{u}^{(0)(m)V}_A \), \( \tilde{u}^{A,(m)V}_A \), \( \tilde{u}^{(0)(m)V}_B \) and \( (\sigma)^{A,(m)V} \) and even for the stress. \( \tilde{u}^{(0)(m)V} \) and \( (\sigma)^{(0)(m)V} \) are \( x_3 \)-independent; \( \tilde{u}^{A,(m)V} \) and \( (\sigma)^{A,(m)V} \) are proportional to the sinusoid \( A_n(x_3) \) or to its spatial derivative \( \partial A_n / \partial x_3 \). Here also, for points \( P_3 \) on the interface, \( \Delta \tilde{u}^{V} \) and \( \Delta (\sigma)^{V} \) are expanded up to terms of first order with respect to \( x_2 = \xi \) in a similar manner as in (A.2) (see Appendix A) for \( \Delta \tilde{u}^{\infty} \) and \( (\Delta \sigma)^{\infty} \). Requiring \( \Delta \tilde{u}^{V} = \Delta \tilde{u}^{\infty} \) and \( (\Delta \sigma)^{V} = (\Delta \sigma)^{\infty} \) lead to the following equations, writing first the conditions corresponding to \( k_3 = 0 \) (i.e. \( \Delta u_3^{(0)V} = \Delta u_3^{(0)\infty} \) and \( \Delta \sigma_{ij}^{(0)V} = \Delta \sigma_{ij}^{(0)\infty} \)).

\[
\Delta u_3^{(0)V} = \Delta u_3^{(0)\infty} \implies \frac{\beta_3^{(2)}}{C_2} + \frac{\beta_3^{(1)}}{C_1} = 0 \tag{a}
\]

\[
\Delta \sigma_{13}^{(0)V} = \Delta \sigma_{13}^{(0)\infty} \text{ and } \Delta \sigma_{23}^{(0)V} = \Delta \sigma_{23}^{(0)\infty} \implies \frac{(1 - \nu_2)\beta_3^{(2)}}{C_2} + \frac{(1 - \nu_1)\beta_3^{(1)}}{C_1} = 0 \tag{b}
\]

\[
(1 - \nu_2)\beta_3^{(2)} - (1 - \nu_1)\beta_3^{(1)} = (Q_c - Q_b) \frac{\text{sgn}(k_1)}{k_1} \tag{c}
\]

where \( C_m = b \mu_m / 2 \pi (1 - \nu_m) \), \( Q_b = i(C_2 - C_1) / 4 \), \( Q_c = i(\nu_2 C_2 - \nu_1 C_1) / 4 \); \( \text{sgn}(k_1) = k_1 / |k_1| \); \( \mu_m \) and \( \nu_m \) are shear modulus and Poisson’s ratio. In equations (12 a to c) above, \( \beta_3^{(m)} \) stands for \( \beta_3^{(m)}(k_1, k_2^{(m)}, k_3 = 0) \). Other elastic field components are zero. The conditions corresponding to \( \Delta u_A^{V} = \Delta u_A^{\infty} \) and \( \Delta \sigma_{ij}^{A,V} = \Delta \sigma_{ij}^{A,\infty} \) are now listed as:

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\[ \Delta u_i^{AV} = \Delta u_i^{AV\infty} \Rightarrow \]
\[ \frac{\Delta u_i^{AV}}{\mu_2} = \frac{\Delta u_i^{AV\infty}}{\mu_1} = \frac{bC_v \varepsilon_n}{8\pi} \frac{k_1}{(k_1^2 + \kappa_n^2)^{3/2}} \]  
(a)

\[ \sqrt{k_1^2 + \kappa_n^2} \left( \frac{\alpha_3^{(2)}}{\mu_2} + \frac{\alpha_3^{(1)}}{\mu_1} \right) + \frac{\bar{\beta}_3^{(2)}}{\mu_2} - \frac{\bar{\beta}_3^{(1)}}{\mu_1} = 0 \]  
(b)

\[ \Delta u_i^{AV} = \Delta u_i^{AV\infty} \Rightarrow (b) \text{ above and} \]
\[ \sqrt{k_1^2 + \kappa_n^2} \left( \frac{\alpha_3^{(2)}}{\mu_2} - \frac{\alpha_3^{(1)}}{\mu_1} \right) + 2 \left( \frac{\bar{\beta}_3^{(2)}}{\mu_2} + \frac{\bar{\beta}_3^{(1)}}{\mu_1} \right) = - \frac{bC_v \varepsilon_n}{8\pi} \frac{k_1}{k_1^2 + \kappa_n^2} \]  
(c)

\[ \Delta u_i^{AV} = \Delta u_i^{AV\infty} \Rightarrow \]
\[ \kappa_n^2 \left( \frac{\alpha_3^{(2)}}{\mu_2} - \frac{\alpha_3^{(1)}}{\mu_1} \right) + 4 \frac{k_1^2 + \kappa_n^2}{\mu_2} \left( \frac{(1 - \nu_2)\bar{\beta}_3^{(2)}}{\mu_2} + \frac{(1 - \nu_1)\bar{\beta}_3^{(1)}}{\mu_1} \right) = - \frac{bC_v \varepsilon_n \kappa_n^2}{8\pi} \frac{k_1}{(k_1^2 + \kappa_n^2)^{3/2}} \]  
(d)

\[ \kappa_n \sqrt{k_1^2 + \kappa_n^2} \left( \frac{\alpha_3^{(2)}}{\mu_2} + \frac{\alpha_3^{(1)}}{\mu_1} \right) + \frac{\bar{\beta}_3^{(2)}}{\mu_2} \left[ \kappa_n^2 + 4(1 - \nu_2)(k_1^2 + \kappa_n^2) \right] \]
\[ - \frac{\bar{\beta}_3^{(1)}}{\mu_1} \left[ \kappa_n^2 + 4(1 - \nu_1)(k_1^2 + \kappa_n^2) \right] = 0 \]  
(e)

\[ \Delta \sigma_{11}^{AV} = \Delta \sigma_{11}^{AV\infty} \Rightarrow \]
\[ k_1^2 \left( \alpha_3^{(2)} - \alpha_3^{(1)} \right) + 2 \sqrt{k_1^2 + \kappa_n^2} \left( \nu_2 \bar{\beta}_3^{(2)} + \nu_1 \bar{\beta}_3^{(1)} \right) = - \frac{iQ_c \varepsilon_n k_1 (k_1^2 + 2\kappa_n^2)}{(k_1^2 + \kappa_n^2)^{3/2}} \]  
(f)

\[ k_1 \sqrt{k_1^2 + \kappa_n^2} \left( \alpha_3^{(2)} + \alpha_3^{(1)} \right) + [(1 + 2\nu_2)k_1^2 + 2\nu_2 \kappa_n^2] \bar{\beta}_3^{(2)} \]
\[ - [(1 + 2\nu_1)k_1^2 + 2\nu_1 \kappa_n^2] \bar{\beta}_3^{(1)} = 0 \]  
(g)

\[ \Delta \sigma_{22}^{AV} = \Delta \sigma_{22}^{AV\infty} \Rightarrow \]
\[ \sqrt{k_1^2 + \kappa_n^2} \left( \alpha_3^{(2)} - \alpha_3^{(1)} \right) + 2(1 - \nu_2) \bar{\beta}_3^{(2)} + 2(2 - \nu_1) \bar{\beta}_3^{(1)} = \frac{i(2Q_c - Q_n) \varepsilon_n k_1}{k_1^2 + \kappa_n^2} \]  
(h)

\[ \sqrt{k_1^2 + \kappa_n^2} \left( \alpha_3^{(2)} + \alpha_3^{(1)} \right) + (3 - 2\nu_2) \bar{\beta}_3^{(2)} - (3 - 2\nu_1) \bar{\beta}_3^{(1)} = 0 \]  
(i)

\[ \Delta \sigma_{33}^{AV} = \Delta \sigma_{33}^{AV\infty} \Rightarrow \]
\[ \kappa_n^2 \left( \alpha_3^{(2)} - \alpha_3^{(1)} \right) + 2 \sqrt{k_1^2 + \kappa_n^2} \left( (2 - \nu_2) \bar{\beta}_3^{(2)} + (2 - \nu_1) \bar{\beta}_3^{(1)} \right) \]

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\[ k_n^2 \sqrt{k_1^2 + \kappa_n^2 (\bar{\alpha}_3^{(2)} - \bar{\alpha}_3^{(1)}) + [(5 - 2\nu_2)\kappa_n^2 + 2(2 - \nu_2)k_1^2] \bar{\beta}_3^{(2)}} - [(5 - 2\nu_1)\kappa_n^2 + 2(2 - \nu_1)k_1^2] \bar{\beta}_3^{(1)} = 0 \] (k)

Next, we are concerned with satisfying boundary conditions: (12) leads to the displacement and stress fields due to an interface straight screw dislocation \((\bar{b}_{ii} = (0,0,b))\) parallel to the \(x_3\) -direction at the origin; the interface is the \(Ox_1x_3\) -plane. (13) provides the complementary terms (to first order in \(\xi_n\)) in the elastic fields of an interfacial sinusoidal screw dislocation.

### III - CALCULATION RESULTS

**III-1. Displacement and stress fields of an interface glide-type sinusoidal edge dislocation**

When \(\xi_n\) is small, the elastic fields (displacement \(\bar{u}^{(m)}\) and stress \(\sigma^{(m)}\)) at an

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arbitrary position \( \bar{x} = (x_1, x_2, x_3) \) may be expressed (to linear terms in \( \xi_n \)) as

\[
\bar{u}^{(m)} = \bar{u}^{(0)(m)} + \bar{u}^{A_n(m)} \\
(\sigma)^{(m)} = (\sigma)^{(0)(m)} + (\sigma)^{A_n(m)},
\]

with \( m = 1 \) and 2 for medium \( S1 \) and \( S2 \), respectively. \( \bar{u}^{(0)(m)} \) and \( (\sigma)^{(0)(m)} \) are of zero order, independent of \( x_3 \), and correspond to the elastic fields of a straight edge dislocation lying on the planar \( OX_1X_3 \)–interface. These are given under continuity requirement of the fields on crossing the interface [10, 11]. \( \bar{u}^{A_n(m)} \) and \( (\sigma)^{A_n(m)} \) are oscillating fields proportional to either \( A_n \) or its spatial derivative \( \partial A_n / \partial x_3 \) with respect to \( x_3 \), written [11] in the forms

\[
\bar{u}^{A_n(m)} = \bar{u}^{A_n(m)} \infty - \bar{u}^{A_n(m)W} \\
(\sigma)^{A_n(m)} = (\sigma)^{A_n(m)} \infty - (\sigma)^{A_n(m)W}.
\]

Here, expressions with \( \infty \) are associated with a sinusoidal edge dislocation in an infinitely extended homogeneous medium \( (m) \) with the equal elastic constants (see oscillating fields given in [5, 6, 12]); second terms with \( W \) read

\[
\bar{u}^{A_n(m)W} = \eta_a^{A_n(m)} \bar{u}_a + \eta_b^{A_n(m)} \bar{u}_b + \eta_c^{A_n(m)} \bar{u}_c + \eta_d^{A_n(m)} \bar{u}_d \\
(\sigma)^{A_n(m)W} = (\sigma)_a^{A_n(m)} + (\sigma)_b^{A_n(m)} + (\sigma)_c^{A_n(m)} + (\sigma)_d^{A_n(m)}
\]

\( \bar{u}^{A_n(m)V} \) and \( (\sigma)^{A_n(m)V} \) are given in [11]; \( \eta_a^{A_n(m)} \) are real determined essentially by continuity conditions of the fields \( \bar{u}^{A_n(m)} \) and \( (\sigma)^{A_n(m)} \) on the interface. These lead to a number of equations denoted by \( e_i^{A_n} \) (see relation (43) in [11]). We follow the treatment in [11] (same definitions and notations) but introduce some changes that follow; in this section, we use the notation \( Eqn \) (N), \( N \) integer, to designate an equation (N) in [11]:

(i) Retaining only terms proportional to \( 1/x_1 \), \( u_2^{A_n(m)} \) is written as

\[
u_{2}^{A_n(m)}(x_1,0,x_3) = -\frac{A_n}{2} e_3^{A_n} \left( \frac{1}{x_1} \right)
\]

were

\[
\begin{align*}
    e_3^{A_n} &\equiv \frac{1}{\mu_m} \left\{ C_m (1 - 2\nu_m) + \eta_a^{A_n(m)} 2\alpha_m^{(m)} + \eta_b^{A_n(m)} (-1)^m (1 - 2\nu_m) 2iQ_b \\
    &+ \eta_c^{A_n(m)} 2i \sum_{i=1}^{3} \alpha_m^{(m)} (-1)^{m-1} 2\rho_m (1 - 2\nu_m) (b_{1c} - b_{2c} + b_{3c}) \right\}
\end{align*}
\]

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Taking into account all the singularity terms with \(-x\) only, we obtain

\[ e_3^{A_3^*:} \text{ constant with } m=1 \text{ and } 2. \]  

We stress that \(e_3^{A_3^*}\) is proportional to the modified Bessel function \(K_n[\kappa_1|x_1|]\) only that has a singularity of the type \(1/x_1\) but additional terms with \(1/x_1\) do exist in \(u_2^{A_2^* (m)}\). Taking into account all the singularity terms with \(1/x_1\) leads to \(e_3^{A_3^*}\) above. (ii) \(e_2^{A_2^*}\) in Eqn (43) is proportional to the modified Bessel function \(K_0[\kappa_2|x_1|]\) only that has a singularity of the type \(\ln|x_1|\); however, there are other terms with \(\ln|x_1|\) in \(\sigma_{12}^{A_2 (m)}\). Collecting all the terms with \(\ln|x_1|\) and setting the coefficient of \(\ln|x_1|\) constant with \(m\) corresponds to \(e_2^{A_2^*}\) constant with \(m\) with

\[
e_2^{A_2^*} = -\nu_m C_m + \eta_n^{A_2 (m)} 2a_{ia}^{(m)} + \eta_n^{A_2 (m)} (-1)^m iQ_b (2 + \nu_m - \rho_m) + \eta_n^{A_2 (m)} 2i \left( \sum_{i=1}^{2} \left( a_{ic}^{(m)} + (-1)^m 2\rho_m c_m (b_{ic} - b_{2c} + b_{ic}) \right) \right) \}
\]

\[
+ \eta_d^{A_2 (m)} 2i \left[ a_{2d}^{(m)} + (-1)^m 2\nu_m F^{(m)} \right].
\]

(18)

\(e_3^{A_3^*}\) is at present considered in place of \(e_2^{A_2^*}\). (iii) We add a new equation \(e_9^{A_9^*}\) corresponding to the condition that \(\sigma_{23}^{A_2 (m)}(x_1,0,x_3)\) is constant with \(m=1\) and 2. Restricting ourselves to all the terms with \(1/x_1\) only, we obtain

\[
\sigma_{23}^{A_2 (m)}(x_1,0,x_3) \equiv \frac{\partial A_2}{\partial x_2} e_9^{A_9^*} \] 

\[
(19)
\]

\[
\text{with}
\]

\[
e_9^{A_9^*} \equiv \nu_m C_m - \eta_n^{A_9 (m)} 2a_{ia}^{(m)} + \eta_n^{A_9 (m)} (-1)^m 2iQ_b \] 

\[
- \eta_d^{A_9 (m)} 2i \left[ \sum_{i=1}^{2} a_{ic}^{(m)} + (-1)^m 2\rho_m c_m (b_{ic} - b_{2c} + b_{ic}) \right] \]

\[
- \eta_d^{A_9 (m)} 2i \left[ a_{2d}^{(m)} + (-1)^m 4\nu_m F^{(m)} \right].
\]

We stress that all the others \(e_i^{A_i}\) in Eqn (43) are unchanged. (iv) In the resolution procedure [11], it is required to choose four equations (\(E_i\), \(i=1\) to 4, corresponding to four independent equations \(e_j^{A_j^*}(1) = e_j^{A_j^*}(2)\). A choice corresponds to Eqn (51); (E1) and (E2), there are at present replaced by (E1)* and (E2)* below.

\((E1)^*:\) \(e_9^{A_9^*}(2) + e_7^{A_7^*}(2) = e_9^{A_9^*}(1) + e_7^{A_7^*}(1)\)

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(E2)*: $e^A_9(2) = e^A_9(1)$ (20)

(E3) and (E4) are unchanged. The identifications corresponding to (E1)* and (E2)*, to be used in place of (E1) and (E2) in Eqn (52), are:

$$
\begin{align*}
 b_1 &= 2i(a_{1d}(2) + 2\nu_1\bar{b}(2))D, \\
 a_{11} &= -2i(a_{1d}(2) + 2\nu_2\bar{b}(2))D_a, \\
 a_{12} &= -iQ_b(\nu_2 - \nu_1)B_b - 2i(a_{1c}(2) + (1 - \theta)a_{3c} + 2\nu_1\nu_2[b_{ic} - 2b_{2c} + (1 + \theta)b_{3c}])C_b \\
 &\quad - 2i(a_{1d}(2) + 2\nu_2\bar{b}(2))D_a + iQ_b(\nu_2 - \nu_1), \\
 a_{13} &= -iQ_b(\nu_2 - \nu_1)B_c - 2i(a_{1c}(2) + (1 - \theta)a_{3c} + 2\nu_1\nu_2[b_{ic} - 2b_{2c} + (1 + \theta)b_{3c}])C_c \\
 &\quad - 2i(a_{1d}(2) + 2\nu_2\bar{b}(2))D_c - 2i(a_{1c}^{(1)} + (1 - \theta)a_{3c}^{(1)} - 2\nu_1\nu_2[b_{ic} - 2b_{2c} + (1 + \theta)b_{3c}]), \\
 a_{14} &= -2i(a_{1d}(2) + 2\nu_2\bar{b}(2))D_d - 2i(a_{1d}(1) - 2\nu_1\bar{b}(1));
\end{align*}
$$

$$
\begin{align*}
 b_2 &= \nu_1C_1 - \nu_2C_2 + 2a_{1a}A + 2i(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}(2))D, \\
 a_{21} &= -2a_{1a}(2)A_a - 2i(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}(2))D_a - 2a_{1a}^{(1)}, \\
 a_{22} &= -2a_{1a}^{(2)}A_b + 2iQ_bB_b - 2i\sum_{s=1}^{3}[a_{2c}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}]C_b \\
 &\quad - 2i(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}(2))D_b - 2iQ_b, \\
 a_{23} &= -2a_{1a}^{(2)}A_c + 2iQ_bB_c - 2i\sum_{s=1}^{3}[a_{2c}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}]C_c \\
 &\quad - 2i(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}(2))D_c + 2i\sum_{s=1}^{3}[a_{2c}^{(1)} + 2\nu_1\nu_2(-1)^{s}b_{sc}], \\
 a_{24} &= -2a_{1a}^{(2)}A_d - 2i(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}(2))D_d + 2i(a_{1d}^{(1)} + a_{2d}^{(1)} - 4\nu_1\bar{b}(1)).
\end{align*}
$$

With changes (i) to (i4) above, the resolution methodology [11] can be followed, step by step, with no more modifications in the various listed relations there (i.e. Eqn (43) to Eqn (56) included, and in addition, all the expressions in the associated Appendix). Again, we stress that the singularity term in $\sigma_{23}^{A,(m)}(x_1,0,x_3)$ is given by (19) above with $e^A_9$ and not Eqn (57) with $e^{A_2}_9$. The solutions $\eta_b^{A,(m)}$ and $\eta_s^{A,(m)}$ ($m=1$ and 2) have been listed in Eqn (53) to Eqn (55). Using the two first equations in Eqn (47), we can write

$$
\eta_{a}^{A,(1)} = \frac{a_{13}(a_{2d}b_1 - a_{1d}b_2) + \left[a_{33}(a_{1d}a_{22} - a_{2d}a_{12}) - a_{32}(a_{1d}a_{23} - a_{2d}a_{13})\right]\eta_{b}^{A,(1)}}{a_{33}(a_{2d}a_{11} - a_{1d}a_{21})},
$$

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\[ \eta_d^{A(1)} = a_{33}(a_2 b_1 - a_1 b_2) + \left[ a_{33}(a_1 a_{22} - a_2 a_{12}) - a_{32}(a_1 a_{23} - a_2 a_{13}) \right]\eta_b^{A(1)} \frac{a_{33}(a_4 a_{21} - a_2 a_{12})}{(a_{33}(a_4 a_{21} - a_2 a_{12}))}. \] (22)

\[ \eta_a^{A(2)} \quad \text{and} \quad \eta_d^{A(2)} \] are obtained using Eqn (44):

\[ \eta_a^{A(2)} = A + A_a \eta_a^{A(1)} + A_b \eta_b^{A(1)} + A_c \eta_c^{A(1)} + A_d \eta_d^{A(1)}, \]

\[ \eta_d^{A(2)} = D + D_a \eta_a^{A(1)} + D_b \eta_b^{A(1)} + D_c \eta_c^{A(1)} + D_d \eta_d^{A(1)}. \] (23)

In summary, all the \( \eta_a^{A(m)} \) \( (m=1 \text{ and } 2) \) have been given, permitting one to express the oscillating parts (15) of the elastic fields \( \vec{u}^{(m)} \) and \( \sigma^{(m)} \) (14). We stress that in our previous work [11], we have imposed coefficients of modified Bessel functions \( K_0[\kappa_n x_i] \) and \( K_1[\kappa_n x_i] \) constant with \( m \) in the elastic fields \( u_2^{A_n(m)}(x_1,0,x_3) \), \( \sigma_{12}^{A_n(m)}(x_1,0,x_3) \) and \( \sigma_{23}^{A_n(m)}(x_1,0,x_3) \) whilst the present study focuses on functions \( \ln|x_i| \) and \( 1/x_i \), respectively. Both procedures are self-consistent and should yield the equal \( \eta_{a d}^{A_n(m)} \).

III-2. Displacement and stress fields of an interface sinusoidal screw dislocation

III-2-1. Displacement and stress fields due to an interface straight screw dislocation

Two distinct values for \( \vec{B}^{(m)} \) are extracted from (12); these are:

\[ \begin{align*}
(a) \quad \vec{B}_{3a}^{(m)} &= \left( \delta_{1m} - \frac{C_2}{C_1} \delta_{2m} \right) \frac{\text{sgn}(k_i)}{k_i^2} = \vec{B}_{3a}^{(m)} \\
(b) \quad \vec{B}_{3b}^{(m)} &= (-1)^{m-1} \frac{(Q_0 - Q_c)}{2(1 - \nu_m)} \frac{\text{sgn}(k_i)}{k_i^2} = \vec{B}_{3b}^{(m)}
\end{align*} \] (24)

where \( \delta_{ij} \) is the Kronecker delta. \( \vec{B}_{3a}^{(m)} \) is obtained using (12 \( a \)) and continuity requirement for \( \vec{u}^{(0)X(m)} \) at the crossing of the interface at \( x_2 = 0 \). \( \vec{B}_{3b}^{(m)} \) is obtained from (12 \( b \) and \( c \)) associated with the stresses. None of these values satisfies the entire (12). The associated elastic fields denoted \( \vec{u}^{(0)X(m)}_{a} \) and \( \sigma^{(0)X(m)}_{a} \) are displayed below. A superposition of these partial fields will provide the complete form of the solution. We have at position \( \vec{x} = (x_1, x_2, x_3) \):
\[ \delta_{ja} u_{3a}^{(0)(m)V} + \delta_{jb} u_{3b}^{(0)(m)V} = \frac{1}{\mu_m} \left( \delta_{ja} (-1)^m V_a^{(m)} + \delta_{jb} 2i(Q_c - Q_b) \right) \tan^{-1} \frac{x_1}{x_2}, \]
\[ \delta_{ja} \sigma_{23a}^{(0)(m)V} + \delta_{jb} \sigma_{23b}^{(0)(m)V} = \left( \delta_{ja} V_a^{(m)} + \delta_{jb} (-1)^m 2i(Q_c - Q_b) \right) \frac{x_1}{r^2}, \]
\[ \delta_{ja} \sigma_{13a}^{(0)(m)V} + \delta_{jb} \sigma_{13b}^{(0)(m)V} = \left( \delta_{ja} (-1)^m V_a^{(m)} + \delta_{jb} 2i(Q_c - Q_b) \right) \left( \frac{x_1}{r^2} + \pi \delta_A(x_3) \delta(x_1) \right) \] (25)

where \( V_a^{(m)} = 4i(1 - \nu_m)(\delta_{1m} - \delta_{2m} C_2 / C_1), j = a \) and \( b \) and \( \delta(x_1) \) is the Dirac delta function; here \( \delta_A \) has the following definition: \( \delta_A(x_2) = 0 \) when \( x_2 \neq 0 \) and \( \delta_A(x_2) = 1 \) when \( x_2 = 0 \); \( r^2 = x_1^2 + x_2^2 \). The other elastic fields are zero. We define the elastic fields \( \vec{u}^{(0)(m)}(\vec{x}) \) and \( (\sigma)_{(0)(m)}^{(0)(m)}(\vec{x}) \) of an interface straight screw dislocation as

\[ \vec{u}^{(0)(m)} = \vec{u}^{(0)(m)\infty} - \vec{u}^{(0)(m)V} \]
\[ (\sigma)^{(0)(m)} = (\sigma)^{(0)(m)\infty} - (\sigma)^{(0)(m)V} \]
with

\[ \vec{u}^{(0)(m)V} = \eta_a^{(m)} u_a^{(0)(m)V} + \eta_b^{(m)} u_b^{(0)(m)V} \]
\[ (\sigma)^{(0)(m)V} = \eta_a^{(m)} (\sigma)_{a}^{(0)(m)V} + \eta_b^{(m)} (\sigma)_{b}^{(0)(m)V} \].

Again \( \vec{u}^{(0)(m)} \) and \( (\sigma)^{(0)(m)} \) are due to a straight screw \( \vec{b}_{II} = (0,0,b) \) parallel to the \( x_3 \) – direction at the origin in an infinite medium (see [5, 6] for example); \( u_a^{(0)(m)V} \) and \( (\sigma)_{a}^{(0)(m)V} \) are given in (25). \( \eta_a^{(m)} \) and \( \eta_b^{(m)} \) are real, to be determined by the condition that the elastic fields satisfy the following relations :

- \( \vec{u}^{(0)(m)}(\vec{x}) \) and \( (\sigma)^{(0)(m)}(\vec{x}) \) are continuous when crossing the \( Ox_1x_3 \) – plane.
- \( \oint_{\Gamma} du_3^{(0)(m)} = b \) for a closed contour \( \Gamma \) in \( x_1x_2 \) encircling the dislocation.
- \( \vec{u}^{(0)(m)V}_a \) vanish far from the interface (i.e. when \( |x_2| \to \infty \)).

The last condition is fulfilled because all the \( \vec{u}^{(0)(m)V}_a \) (25) vanish when \( |x_2| \to \infty \). Also \( (\sigma)^{(0)(m)} \) and \( (\sigma)^{(0)(m)V}_a \) vanish at infinity. Under such conditions, \( \vec{u}^{(0)(m)}(\vec{x}) \) and \( (\sigma)^{(0)(m)}(\vec{x}) \) do correspond to an interface straight screw dislocation. Next, we express the quantities involved in the above mentioned requirements and proceed to satisfy these.

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\[ u_{3}^{(0)(1)} = u_{3}^{(0)(2)} \text{ and } \int_{\Gamma} du_{3}^{(0)(m)} = b \implies \]
\[ \left( \eta_{a}^{(m)(-1)m}V_{a}^{(m)} + \eta_{b}^{(m)(-1)m}V_{b}^{(m)} \right) \mu_{m} \equiv \epsilon_{1}' \]
\[ \sigma_{13}^{(0)(1)} = \sigma_{13}^{(0)(2)} \implies \eta_{a}^{(m)(-1)m}V_{a}^{(m)} + \eta_{b}^{(m)(-1)m}V_{b}^{(m)} \equiv \epsilon_{2}' \]
\[ \sigma_{23}^{(0)(1)} = \sigma_{23}^{(0)(2)} \implies D_{m} - \eta_{a}^{(m)(-1)m}V_{a}^{(m)} + \eta_{b}^{(m)(-1)m}V_{b}^{(m)} \equiv \epsilon_{3}' \]

where \( D_{m} = b \mu_{m} / 2\pi = C_{m} (1 - \nu_{m}) \); all \( \epsilon_{i}' \) are constant with \( m=1 \) and 2. Only expressions interconnecting the \( \eta_{a}^{(m)} \) and \( \eta_{b}^{(m)} \) may be derived from (28). But only one expression leaves unchanged elastic fields \( \vec{u}^{(0)(m)} \) and \( (\sigma)^{(0)(m)} \) by inverting the elastic constants. This is written as

\[ \eta_{a}^{(m)(-1)m}V_{a}^{(m)} + \eta_{b}^{(m)(-1)m}V_{b}^{(m)} \equiv \frac{(\mu_{1}\delta_{1m} + \mu_{2}\delta_{2m})(D_{2} - D_{1})}{\mu_{1} + \mu_{2}} , \quad m=1 \text{ and } 2. \quad (29) \]

It may be obtained using \( \epsilon_{1}' \) and \( \epsilon_{3}' \) in the form

\[ D_{m} + (-1)^{m-1}\mu_{m}\epsilon_{1}(m) = \epsilon_{3}(m) ; \]

then, taking \( m=2 \) and introducing \( \epsilon_{1} \) and \( \epsilon_{3} \) (using (28)) in place of \( \epsilon_{1}' \) and \( \epsilon_{3}' \), we obtain (29) for \( m=1 \). The case \( m=2 \) in (29) is obtained in a similar manner.

With (29), the elastic fields are:

\[ u_{3}^{(0)(m)}(\vec{x}) = \frac{b}{2\pi} \tan^{-1} \frac{x_{z}}{x_{1}} + \frac{D_{1} - D_{2}}{\mu_{1} + \mu_{2}} \tan^{-1} \frac{x_{1}}{x_{2}} , \]
\[ \sigma_{23}^{(0)(m)}(\vec{x}) = \frac{b\mu_{1}\mu_{2}}{\pi(\mu_{1} + \mu_{2})r^{2}} x_{1} , \]
\[ \sigma_{13}^{(0)(m)}(\vec{x}) = -\frac{b\mu_{1}\mu_{2}}{\pi(\mu_{1} + \mu_{2})r^{2}} x_{2} + \delta_{A}(x_{2}) \frac{\mu_{a}(D_{1} - D_{2})}{\mu_{1} + \mu_{2}} \pi\delta(x_{1}) . \quad (30) \]

These are in complete agreement with previous works (see [13] and references therein). Other elastic field components are zero.

### III-2-2. Elastic fields of an interface sinusoidal screw dislocation

Five values for \( (\alpha_{3}^{(m)}(\kappa_{n}), \beta_{3}^{(m)}(\kappa_{n})) \) are extracted from (13); these are

P.N.B. ANONGBA
\[
\overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1a}^{(m)}}{2} \left( \frac{k_i}{\sqrt{k_i^2 + \kappa_n^2}} \right)^{3/2} \equiv \overline{\alpha}_{3a}^{(m)}, \quad \overline{\beta}_3^{(m)} = 0 \equiv \overline{\beta}_{3a}^{(m)}; \\
\overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1b}^{(m)}}{\sqrt{\kappa_i^2 + \kappa_n^2}} \left( 4 - \nu_m - 3\rho_m - 2(1 - \rho_m) \right) \equiv \overline{\alpha}_{3b}^{(m)}, \\
\overline{\beta}_3^{(m)} = \frac{\xi_n r_{1b}^{(m)}}{\sqrt{k_i^2 + \kappa_n^2}} \left( - \frac{1}{k_i^2 + \kappa_n^2} \right) \equiv \overline{\beta}_{3b}^{(m)}; \\
\overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1c}^{(m)}}{\kappa_n^2} \left( (2 - \nu_m)(1 + Q_r) \sqrt{\frac{k_i^2 + \kappa_n^2}{k_i}} - \frac{2(2 - \nu_m)\kappa_n^2}{k_i^2 + \kappa_n^2} \right) \\
- \frac{2k_i}{\sqrt{k_i^2 + \kappa_n^2}} + \frac{\kappa_n^2 k_i}{\left( k_i^2 + \kappa_n^2 \right)^{3/2}} \equiv \overline{\alpha}_{3c}^{(m)} \\
\overline{\beta}_3^{(m)} = \frac{\xi_n r_{1c}^{(m)}}{k_i} \left( 1 + Q_r - \frac{2\kappa_n^2}{k_i^2 + \kappa_n^2} \right) \equiv \overline{\beta}_{3c}^{(m)} \text{ independent of } m; \\
\overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1d}^{(m)}}{\left( k_i^2 + \kappa_n^2 \right)^{3/2}} \left( \frac{s_{2d}^{(m)}}{k_i^2 + \kappa_n^2} - \frac{\kappa_n^2 s_{3d}^{(m)}}{k_i^2 - s_m \kappa_n^2} - \frac{\kappa_n^2 s_{4d}^{(m)}}{k_i^2 - r_m \kappa_n^2} \right) \equiv \overline{\alpha}_{3d}^{(m)}, \\
\overline{\beta}_3^{(m)} = \frac{\xi_n r_{1d}^{(m)}}{k_i} \left( \frac{r_m + \tilde{Q}_r}{k_i^2 - r_m \kappa_n^2} + \frac{1 - \tilde{Q}_r}{k_i^2 + \kappa_n^2} \right) \equiv \overline{\beta}_{3d}^{(m)}; \\
\overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1e}^{(m)}}{\sqrt{k_i^2 + \kappa_n^2}} \left( \frac{s_{3e}^{(m)}}{k_i^2 + \kappa_n^2} + \frac{s_{2e}^{(m)} - s_{3e}^{(m)}}{k_i^2 + \kappa_n^2} \right) \equiv \overline{\alpha}_{3e}^{(m)}, \\
\overline{\beta}_3^{(m)} = \frac{\xi_n r_{1e}^{(m)}}{k_i} \left( \frac{-1}{k_i^2 + \kappa_n^2} + \frac{2}{k_i^2 + \kappa_n^2} \right) \equiv \overline{\beta}_{3e}^{(m)} \quad (31)
\]

where
\[s_{1a}^{(m)} = (-1)^m \mu_m bC_v / 8\pi, \quad C_v = \left[ 1/(1 - \nu_1) - 1/(1 - \nu_2) \right], \quad Q_r = Q_c / Q_b = 1 / \tilde{Q}_r, \]
\[r_m = V_m / (1 - 2\nu_m), \quad \rho_m = \nu_1 \delta_{m1}^2 + \nu_2 \delta_{m1}, \quad s_m = \rho_m / (1 - 2\rho_m), \]
\[\Omega_{1b} = [(1 - \nu_1)(1 - 2\nu_2) + (1 - \nu_2)(1 - 2\nu_1)] / 4(1 - \nu_1)(1 - \nu_2); \]
\[s_{1b}^{(m)} = (-1)^{m-1} C_v D_m / 8(2 - \nu_1 - \nu_2), \quad r_{1b}^{(m)} = C_v C_m / 16(1 - \Omega_{1b}); \]
\[s_{1c}^{(m)} = (-1)^m iQ_b / 2, \quad r_{1c} = -iQ_b / 4; \]
\[s_{1d}^{(m)} = (-1)^m iQ_c / 2; \]
\[s_{1e}^{(m)} = -2\tilde{Q}_r + 2[\rho_m - V_m - 2\nu_m (1 - 2\rho_m)] / (1 - 2\nu_m)(1 - 2\rho_m), \]
\[s_{2d}^{(m)} = (s_m + \tilde{Q}_r) / (1 - 2\rho_m), \quad s_{4d}^{(m)} = (r_m + \tilde{Q}_r)(5 - 4\nu_m) / (1 - 2\nu_m), \]

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None of these couples satisfies the entire conditions (13). For each couple, we give in Appendix B the associated oscillating elastic fields \( \vec{u}_A_n(m) = \vec{u}_A_n(m) + \vec{u}_A_n(m)W \) and \( (\sigma)_{A_n(m)} = (\sigma)_{A_n(m)}A + (\sigma)_{A_n(m)}B \) defined in (9). A superposition of these partial fields will provide the complete form of solution (to first order in \( \xi_n \)). The elastic fields \( \vec{u}(m)(\vec{x}) \) and \( (\sigma)(m)(\vec{x}) \) of an interface sinusoidal screw dislocation may be written as

\[
\vec{u}(m) = \vec{u}(0)^{(m)} + \vec{u}_A_n(m) ; \\
(\sigma)(m) = (\sigma)(0)^{(m)} + (\sigma)_A_n(m) ;
\]

(33)

\( \vec{u}(0)^{(m)} \) and \( (\sigma)(0)^{(m)} \) (30) correspond to the fields of a straight screw dislocation lying on a planar interface; \( \vec{u}_A_n(m) \) and \( (\sigma)_A_n(m) \) are oscillating expressions proportional to either the sinusoid \( A_n(x_3) \) or its spatial derivative \( \partial A_n/\partial x_3 \) in the forms

\[
\vec{u}_A_n(m) = \vec{u}_A_n(m)A - \vec{u}_A_n(m)W ; \\
(\sigma)_A_n(m) = (\sigma)_A_n(m)A - (\sigma)_A_n(m)W ;
\]

(34)

with

\[
\vec{u}_A_n(m)W = \sum_{j=0}^{\infty} \eta_j^{A_n(m)} \vec{u}_j^{A_n(m)}W \\
(\sigma)_A_n(m)W = \sum_{j=0}^{\infty} \eta_j^{A_n(m)} (\sigma)_j^{A_n(m)}W .
\]

(35)

\( \vec{u}_A_n(m)A \) and \( (\sigma)_A_n(m)A \) are given in Appendix B (for \( \vec{u}_A_n(m)A \) and \( (\sigma)_A_n(m)A \), see [5, 6]) ; \( \eta_j^{A_n(m)} \) are real to be determined by the requirement that the elastic fields be continuous when crossing the interface. It is sufficient to write this condition for points on the average interface plane. Before displaying the corresponding equations, we stress what follows.

- Both \( u^{A_n(m)}_{1}(x_1, 0, x_3) \) and \( \sigma^{A_n(m)}_{13}(x_1, 0, x_3) \) contain two terms with singularities \( \ln|x_1| \) and \( 1/x_1^2 \); the associated coefficients \( e^{A_n}_{11} \) and \( e^{A_n}_{81} \)

\[P.N.B. \ ANONGBA\]
and \((e_{12}^A\) and \(e_{52}^A\), respectively, have been taken constant.

- \(u_2^{A_2(m)}(x_1,0,x_3)\), \(\sigma_{12}^{A_2(m)}(x_1,0,x_3)\) and \(\sigma_{23}^{A_2(m)}(x_1,0,x_3)\) are bounded functions. Under such conditions, we have considered their linear forms with respect to \(x_1\) and posed the terms proportional to \(x_1\) constant with \(m=1\) and \(2\).

- \(\sigma_{11}^{A_2(m)}(x_1,0,x_3)\) and \(\sigma_{22}^{A_2(m)}(x_1,0,x_3)\) have terms with singularities \(1/x_1\) and those of \(\partial^2 I_0/\partial |x_1|^2 \equiv (2/x_1^2 + \kappa^2_n \ln \kappa_n |x_1|)/x_1\); the associated coefficients \((e_{41}^A\) and \(e_{51}^A\)\) and \((e_{42}^A\) and \(e_{52}^A\), respectively, have been set constant.

- \(u_3^{A_3(m)}(x_1,0,x_3)\) and \(\sigma_{33}^{A_3(m)}(x_1,0,x_3)\) contain terms with the singularity \(1/x_1\) only. The associated coefficients \(e_{3}^A\) and \(e_{6}^A\) are taken constant.

We may write:

\[
\frac{1}{\mu_m} \left \{ C_m (1 - 2H_m) - \eta_d^{A_2(m)} \frac{S_{12}^{(m)}}{2} + \eta_b^{A_2(m)} \frac{S_{12}^{(m)}}{2} + \eta_c^{A_2(m)} s_{1c}^{(m)} (1 - 2H_m) \right \} = e_{12}^A,
\]

\[
\frac{1}{\mu_m} \eta_c^{A_2(m)} \left [(2 - H_m) Q_r - H_m \right ] = e_{12}^A,
\]

\[
u_1^{A_2(1)} = u_2^{A_2(2)} \Rightarrow
\]

\[
\frac{1}{\mu_m} \left \{ \eta_a^{A_2(m)} (-1)^m S_{1a}^{(m)} / 2 - \eta_b^{A_2(m)} (-1)^m S_{1b}^{(m)} \left [ 2(1 - H_m) - \sqrt{\Omega_{1b}} (4 - H_m - 3 H_m) \right ] + r_{1b}^{(m)} (1 - \sqrt{\Omega_{1b}}) \right \} + \eta_d^{A_2(m)} \left \{ (-1)^m S_{1d}^{(m)} \left [ S_{2d}^{(m)} - S_{3d}^{(m)} \right ] + r_{1d}^{(m)} (1 - \tilde{Q}_r) \right \} + \eta_e^{A_2(m)} \left \{ 2r_{1e}^{(m)} + (-1)^m S_{1e}^{(m)} \left [ S_{2e}^{(m)} - S_{3e}^{(m)} \right ] \right \} = e_{2}^A;
\]

\[
u_3^{A_2(1)} = u_3^{A_2(2)} \Rightarrow
\]

\[
\frac{1}{\mu_m} \left \{ D_m / 2 - \eta_c^{A_2(m)} s_{1c}^{(m)} \left [ - 2 + H_m (1 + Q_r) \right ] + \eta_d^{A_2(m)} (-1)^{m-1} 4(1 - H_m) R_{1d}^{(m)} \right \} = e_{3}^A;
\]
\[ \sigma_{11}^{A_1} = \sigma_{11}^{A_2} \Rightarrow \]
\[ \frac{C_m}{4} - \eta_a^{A(m)} \left( s_{1a}^{(m)} + \frac{n a^{A(m)} s_{1a}^{(m)}}{2} + \eta_c^{A(m)} s_{lc}^{(m)} \right) \left[ 1 - \nu_m (1 - Q_r) \right] \]
\[ - \eta_d^{A(m)} \left[ s_{1d}^{(m)} + s_{2d}^{(m)} + (-1)^m 2 \nu_m (1 + r_m) r_{1d}^{(m)} \right] - \eta_e^{A(m)} \left[ s_{le}^{(m)} + s_{2e}^{(m)} + (-1)^m 2 \nu_m r_{le}^{(m)} \right] \equiv e_{41}^{A_i}, \]
\[ \eta_c^{A(m)} s_{lc}^{(m)} [(v_m - 2) Q_r + \nu_m] \equiv e_{42}^{A_i}; \]
\[ \sigma_{22}^{A_1} = \sigma_{22}^{A_2} \Rightarrow \]
\[ \frac{C_m}{4} (1 - 2 \nu_m) - \eta_a^{A(m)} \left( s_{1a}^{(m)} + \frac{n a^{A(m)} s_{1a}^{(m)}}{2} - \eta_c^{A(m)} s_{lc}^{(m)} \right) \left[ 1 - 2 \nu_m + Q_r \right] \]
\[ - \eta_d^{A(m)} \left[ s_{1d}^{(m)} + s_{2d}^{(m)} + (-1)^m 2 \nu_m (1 + r_m) r_{1d}^{(m)} \right] \]
\[ - \eta_e^{A(m)} \left[ s_{le}^{(m)} + s_{2e}^{(m)} + (-1)^m 2 \nu_m r_{le}^{(m)} \right] \equiv e_{51}^{A_i}, \]
\[ \eta_c^{A(m)} s_{lc}^{(m)} [(2 - \nu_m) Q_r - \nu_m] \equiv e_{52}^{A_i}; \]
\[ \sigma_{33}^{A_1} = \sigma_{33}^{A_2} \Rightarrow \]
\[ \frac{C_m}{2} + \eta_c^{A(m)} 2 s_{lc}^{(m)} + \eta_d^{A(m)} (-1)^{m-1} 2 (2 - \nu_m) (1 + r_m) r_{1d}^{(m)} \]
\[ + \eta_e^{A(m)} (-1)^{m-1} 2 (2 - \nu_m) r_{le}^{(m)} \equiv e_{6}^{A_i}; \]
\[ \sigma_{12}^{A_1} = \sigma_{12}^{A_2} \Rightarrow \]
\[ \eta_a^{A(m)} (-1)^{m-1} \frac{s_{1a}^{(m)}}{2} + \eta_b^{A(m)} \left( (-1)^m s_{1b}^{(m)} \right) \left[ 2(1 - \rho_m) - \Omega_{lb} (4 - \nu_m - 3 \rho_m) \right] \]
\[ + \eta_c^{A(m)} \left( (-1)^m s_{1c}^{(m)} \right) \left[ s_{2d}^{(m)} - s_{3d}^{(m)} + s_{4d}^{(m)} \right] + \eta_{lb}^{(m)} \left( (1 + r_m) (r_m - 1 + Q) \right) \]
\[ + \eta_{le}^{A(m)} \left( (-1)^m s_{le}^{(m)} \right) \left[ s_{3e}^{(m)} - s_{2e}^{(m)} \right] - 2 r_{le}^{(m)} \equiv e_{7}^{A_i}; \]
\[ \sigma_{13}^{A_1} = \sigma_{13}^{A_2} \Rightarrow \]
\[ - \frac{C_m}{4} - \eta_a^{A(m)} \left( s_{1a}^{(m)} + \frac{n a^{A(m)} s_{1a}^{(m)}}{2} - \eta_c^{A(m)} s_{lc}^{(m)} \right) \left[ s_{1e}^{(m)} + s_{2e}^{(m)} + (-1)^{m-1} 4 (1 - \nu_m) r_{le}^{(m)} \right] \]
\[ - \eta_d^{A(m)} \left[ s_{1d}^{(m)} + s_{2d}^{(m)} + (-1)^m 2 (1 - \nu_m) (1 + r_m) (1 - r_m - Q_r) r_{1d}^{(m)} \right] \equiv e_{81}^{A_i}, \]
\[ - \frac{D_m}{4} + \eta_c^{A(m)} s_{lc}^{(m)} (Q_r - 1) + \eta_d^{A(m)} (-1)^m 2 (1 - \nu_m) (1 + r_m) r_{1d}^{(m)} \]
\[ + \eta_e^{A(m)} (-1)^m 2 (1 - \nu_m) r_{le}^{(m)} \equiv e_{82}^{A_i}; \]
\[ \sigma_{23}^{A_1} = \sigma_{23}^{A_2} \Rightarrow \]

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\[
\eta_{a}^{A_{i}(m)} \left( -1 \right)^{m} \frac{S_{1a}^{(m)}}{2} - \eta_{b}^{A_{i}(m)} \left( -1 \right)^{m} \frac{S_{1b}^{(m)}}{2} \left[ 2(1 - \rho_{m}) - \sqrt{\Omega_{1b}}(4 - \nu_{m} - 3\rho_{m}) \right] + r_{lb}^{(m)} \left[ 1 - \sqrt{\Omega_{1b}} - 2(1 - \nu_{m})(1 - \Omega_{1b})\sqrt{\Omega_{1b}} \right] \\
+ \eta_{d}^{A_{i}(m)} \left( -1 \right)^{m} \frac{S_{1d}^{(m)}}{2} \left[ S_{2d}^{(m)} - S_{3d}^{(m)} + S_{4d}^{(m)} + r_{ld}^{(m)}(1 - \widetilde{Q}_{r}) \right] + \eta_{e}^{A_{i}(m)} \left( -1 \right)^{m} \frac{S_{1e}^{(m)}}{2} \left[ S_{2e}^{(m)} - S_{3e}^{(m)} + r_{le}^{(m)} \right] = e_{s}^{A_{i}}. 
\] 

(36)

All \( e_{i}^{A_{i}}(m) \) are constant with \( m = 1 \) and \( 2 \) (i.e. \( e_{i}^{A_{i}}(1) = e_{i}^{A_{i}}(2) \)). We can see that \( e_{42}^{A_{i}} = -e_{52}^{A_{i}} = -\mu_{m}e_{12}^{A_{i}} \). There are twelve equations in (36) with ten unknowns \( \eta_{a}^{A_{i}(m)} \). A solution can be found with ten independent equations. A methodology of the solution may be [11]: (i) express the \( \eta_{i}^{A_{i}(2)} \) as a function of the \( \eta_{i}^{A_{i}(1)} \) giving five relations, (ii) report these relations in five independent equations in (36); we have then a linear system of five equations with unknowns \( \eta_{i}^{A_{i}(1)} \) that can be resolved by the usual classical method with determinants. The result is five expressions linking the \( \eta_{i}^{A_{i}(1)} \) with the elastic constants of the mediums \( m = 1 \) and \( 2 \). (iii) Come back to the \( \eta_{i}^{A_{i}(2)} \) relations to find their respective values as a function of the elastic constants. Here however, we shall proceed differently by first calculating values of a number of \( e_{i}^{A_{i}} \), including those associated with the singularity \( 1/x_{i} \) in the stress fields. When a crack is represented by a continuous distribution of infinitesimal dislocations, stress terms that have a singularity \( 1/x_{i} \) contribute a non-zero value to the crack extension force. We have, using (36),

\[
\nu_{m} C_{m} / 2 + (-1)^{m} 2(1 - 2\nu_{m}) \left[ \eta_{c}^{A_{i}(m)} (1 + Q_{r}) r_{c}^{(m)} + \eta_{d}^{A_{i}(m)} (1 + r_{m}) r_{ld}^{(m)} + \eta_{e}^{A_{i}(m)} r_{le}^{(m)} \right] = e_{42}^{A_{i}} + e_{41}^{A_{i}} - e_{51}^{A_{i}} \equiv E_{1}^{A_{i}}. 
\] 

(37)

Equations \( e_{3}^{A_{i}}(1) = e_{3}^{A_{i}}(2) \), \( e_{6}^{A_{i}}(1) = e_{6}^{A_{i}}(2) \), \( E_{1}^{A_{i}}(1) = E_{1}^{A_{i}}(2) \) and \( e_{42}^{A_{i}}(1) = e_{42}^{A_{i}}(2) \) are four relations that provide the values of unknowns \( \eta_{c}^{A_{i}(m)} \) and \( \eta_{d}^{A_{i}(m)}(1 + r_{m}) r_{ld}^{(m)} + \eta_{e}^{A_{i}(m)} r_{le}^{(m)} \), \( m = 1 \) and \( 2 \), in the forms:

\[
e_{52}^{A_{i}} = \eta_{c}^{A_{i}(m)} s_{s}^{(m)} [2(1 - \nu_{m})Q_{r} - \nu_{m}] \\
= \frac{1}{3(\Gamma - 1)(\nu_{2} - \nu_{1})} \left\{ (\nu_{2} C_{2} - \nu_{1} C_{1})[(1 - \nu_{2})(2 - \nu_{1}) - (1 - \nu_{1})(2 - \nu_{2})] \right\}
\]

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\[ + \left( C_2 - C_1 \right) \left[ (1 - \nu_2)(1 - 2\nu_1) - \Gamma(1 - \nu_1)(1 - 2\nu_2) \right], \]

\[ (-1)^m \left[ \eta_d^{A(m)} (1 + r_m) r_{id}^{(m)} + \eta_e^{A(m)} r_{ie}^{(m)} \right] = \]

\[ \frac{2 - \rho_m (v_2 C_2 - v_1 C_1) + (1 - 2\rho_m) (C_2 - C_1)}{3(v_2 - v_1)} \] + \[ 2(1 + \varrho_s) \eta_{c}^{A(m)} s_{ie}^{(m)} \]

where \( \Gamma = \mu_2 / \mu_1 \). We also have (see (36))

\[ \eta_{c}^{A(m)} s_{ie}^{(m)} (Q_e - 1) + (-1)^m 2(1 - \nu_m) \left[ \eta_d^{A(m)} (1 + r_m) r_{id}^{(m)} + \eta_e^{A(m)} r_{ie}^{(m)} \right] = \mu_m e_{11}^{A} - e_{51}^{A}. \]  

(39) with \( m=1 \) and 2 is a system of two equations with unknowns \( e_{11}^{A} \) and \( e_{51}^{A} \) that can be solved with the help of the second of (38). A direct inspection of various \( e_i^{A} \) in (36) shows that these can be given values with the help of (38). Hence, we display following expressions \( e_{52}^{A} \) is given in (38)) :

\[ e_{11}^{A} = \frac{1}{6(\mu_2 - \mu_1)} \left\{ (\nu_1 C_1 - \nu_2 C_2) [(1 - \nu_1)(2 - \nu_2) - (1 - \nu_2)(2 - \nu_1)] \right. \]

\[ + (C_1 - C_2) [(1 - \nu_1)(1 - 2\nu_2) - (1 - \nu_2)(1 - 2\nu_1)] \],

\[ e_{51}^{A} = -e_{52}^{A} / 2, \]

\[ e_{3}^{A} = \frac{b}{4\pi} + \frac{1 - \nu_1)(1 - \nu_2)(\nu_1 C_1 - \nu_2 C_2)}{\mu_1 (1 - \nu_2)(1 - 2\nu_1) - \mu_2 (1 - \nu_1)(1 - 2\nu_2)} \]

\[ + \frac{(1 - \nu_2)(1 - 2\nu_2) - (1 - \nu_1)(1 - 2\nu_1)}{\mu_1 (1 - \nu_2)(1 - 2\nu_1) - \mu_2 (1 - \nu_1)(1 - 2\nu_2)} e_{52}^{A}, \]

\[ e_{41}^{A} = \frac{\nu_1 C_1 (1 - \nu_1)(1 - 2\nu_2) \Gamma - \nu_2 C_2 (1 - \nu_2)(1 - 2\nu_1)}{2[\Gamma(1 - \nu_1)(1 - 2\nu_2) - (1 - \nu_2)(1 - 2\nu_1)]} \]

\[ + \frac{\nu_1 (1 - 2\nu_2) \Gamma - \nu_2 (1 - 2\nu_1)}{2[\Gamma(1 - \nu_1)(1 - 2\nu_2) - (1 - \nu_2)(1 - 2\nu_1)]} e_{52}^{A}, \]

\[ e_{6}^{A} = \frac{\Gamma C_1 - C_2}{2(\Gamma - 1)} + \frac{\Gamma (\nu_1 C_1 - \nu_2 C_2) [(1 - \nu_1)(2 - \nu_2) - (1 - \nu_2)(2 - \nu_1)]}{2(\Gamma - 1)[\Gamma(1 - \nu_1)(1 - 2\nu_2) - (1 - \nu_2)(1 - 2\nu_1)]} \]

\[ + \frac{\nu_1 (1 - 2\nu_2) \Gamma - \nu_2 (1 - 2\nu_1)}{2[\Gamma(1 - \nu_1)(1 - 2\nu_2) - (1 - \nu_2)(1 - 2\nu_1)]} e_{52}^{A}. \]  

(40)

These expressions are unchanged by inverting the elastic constants. At this stage, only \( \eta_{c}^{A,(m)} \) is given a value (38). To proceed further, we introduce two parameters
\[ \eta^{A_{(m)}}_{b+a} = \eta^{A_{(m)}}_b + \eta^{A_{(m)}}_a, \]
\[ \eta^{A_{(m)}}_{b-a} = \eta^{A_{(m)}}_b - \eta^{A_{(m)}}_a, \]

(41)

and search for these two parameters and \( \eta^{A_{(m)}}_d \) and \( \eta^{A_{(m)}}_e \), \( m = 1 \) and \( 2 \); we have eight unknowns and look for eight independent equations. Because the value of \( \epsilon^{A_{11}} \) is known (40), its expressions with the associated \( \eta^{A_{(m)}}_1 \) (36) provide two equations; two additional relations are given by the second of (38) with \( \eta^{A_{(m)}}_d (1 + r_m) r^{(m)}_{id} + \eta^{A_{(m)}}_e r^{(m)}_{ie} \). These first four equations can be written in the form

\[ c_{11}^{(m)} \eta^{A_{(m)}}_d + c_{12}^{(m)} \eta^{A_{(m)}}_e = b_1^{(m)}, \]
\[ c_{21}^{(m)} \eta^{A_{(m)}}_d + c_{22}^{(m)} \eta^{A_{(m)}}_e = b_2^{(m)} \]

(42)

where

\[ c_{11}^{(m)} = -c_{1d}^{(m)} s_{2d}, \quad c_{12}^{(m)} = -c_{1e}^{(m)} s_{2e}, \]
\[ b_1^{(m)} = -\eta^{A_{(m)}}_{b-a} s_{1a}^{(m)} + \mu_m e_{11}^{A_{(m)}} - \frac{C_m (1 - 2 \nu_m)}{4} - \eta^{A_{(m)}}_e s_{1e}^{(m)} (1 - 2 \nu_m), \]
\[ c_{21}^{(m)} = (1 + r_m) r^{(m)}_{id}, \quad c_{22}^{(m)} = r^{(m)}_{ie}, \]
\[ b_2^{(m)} = \frac{(-1)^m}{4} \left( (2 - \rho_m)(\nu_2 C_2 - \nu_1 C_1) + (1 - 2 \rho_m)(C_2 - C_1) \right) \]
\[ + (1 + Q_r) \eta^{A_{(m)}}_e s_{1e}^{(m)} \).

We obtain

\[ \eta^{A_{(m)}}_e = \frac{c_{21}^{(m)} b_1^{(m)} - c_{11}^{(m)} b_2^{(m)}}{c^{(m)}_{21} c_{12}^{(m)} - c_{11}^{(m)} c_{22}^{(m)}}, \]
\[ \eta^{A_{(m)}}_d = \frac{c_{22}^{(m)} b_1^{(m)} - c_{12}^{(m)} b_2^{(m)}}{c^{(m)}_{22} c_{11}^{(m)} - c_{12}^{(m)} c_{21}^{(m)}}. \]

(43)

Relations (43) give \( \eta^{A_{(m)}}_d \) and \( \eta^{A_{(m)}}_e \), \( m = 1 \) and \( 2 \), as a function of \( \eta^{A_{(m)}}_{b-a} \) only through \( b_1^{(m)} \). It remains to evaluate \( \eta^{A_{(m)}}_{b+a} \) and \( \eta^{A_{(m)}}_{b-a} \) with four independent equations. We may use : \( \epsilon^{A_{22}}_2 (1) = \epsilon^{A_{22}}_2 (2), \quad \epsilon^{A_{77}}_7 (1) = \epsilon^{A_{77}}_7 (2), \)
\( \epsilon^{A_{22}}_0 (1) + \epsilon^{A_{22}}_2 (1) = \epsilon^{A_{22}}_2 (2) + \epsilon^{A_{22}}_2 (2) \) and \( \epsilon^{A_{88}}_8 (1) = \epsilon^{A_{88}}_8 (2) \). We then have four independent equations with unknown \( \eta^{A_{(m)}}_{b,a} \) that can be resolved by the usual classical method with determinants (see [11] for analogous analysis). In summary, the elastic fields \( (\mathbf{u}^{(m)} \) and \( (\sigma)_{(m)} : (33)) \) of an interfacial sinusoidal

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screw dislocation (Figure 2) to linear order in $\xi_n$, are the sum of two terms: the first ones correspond to the elastic fields ($\vec{u}^{(0)(m)}$ and $\sigma^{(0)(m)}$: (30)) produced by an interface straight screw dislocation; second terms are oscillating expressions ($\vec{u}^{A_i(m)}$, $\sigma^{A_i(m)}$: (34)), proportional to the perturbation $A_n(x_3) = \xi_n \sin \kappa_n x_3$ or its partial derivative $\partial A_n / \partial x_3$. These latter expressions can be written as linear combinations (35) of partial elastic fields $\vec{u}^{A_i(m)} (i = a$ to $e)$ (see Appendix B). The associated proportionality coefficients $\eta_i^{A_i(m)}$ fulfilled continuity requirements (36) of the elastic ($\vec{u}^{(m)}$, $\sigma^{(m)}$) on crossing the sinusoidal interface. These can be solved to provide values for the $\eta_i^{A_i(m)}$ (see (38) and (43) with the associated text). A number of $e_i^{A_i}$ values have been calculated (40) that include the coefficients of the various stress terms with the singularity $1/x_1$. We shall make use of certain terms in the crack analysis to come (part II of this study).

IV - DISCUSSION AND CONCLUDING REMARKS

In the present study, the displacement and stress fields of sinusoidal dislocations (glide-type edge and screw) lying, at the origin, on a non-planar interface $S$ with the form of a corrugated sheet (Figure 2), have been determined. Stress terms with the singularity $1/x_1$ have been emphasized in view of crack analyses. As an illustration, consider the stress $\sigma^{(m)}_{33}$ (case of sinusoidal screw dislocation only): on the interface $x_2 = A_n$, is assumed small. We can take the linear form of the stress up to term with $x_2$, i.e.

$$\sigma^{(m)}_{33}(x_1, x_2, x_3) = \sigma^{(m)}_{33}(x_1, 0, x_3) + \frac{\partial \sigma^{(m)}_{33}(x_1, 0, x_3)}{\partial x_2} x_2.$$

We have

$$\sigma^{(m)}_{33}(x_1, 0, x_3) = \sigma^{(0)(m)}_{33}(x_1, 0, x_3) + \sigma^{A_i(m)}_{33}(x_1, 0, x_3).$$

The first term $\sigma^{(0)(m)}_{33}$ (30) is zero. Restricting ourselves to stress terms with $1/x_1$ only, it is easy to see that

$$\sigma^{A_i(m)}_{33}(x_1, 0, x_3) = -\frac{\partial A_n}{\partial x_3} \frac{4 e_i^{A_i}}{x_1}.$$  (44)
The value of $e_6^{A_n}$ is known (40) with no reference to the $\eta_i^{A_n(m)}$. This result is sufficient in the analysis of a number of special crack fronts when $\sigma_{33}^{(m)}$ is involved (see [5, 6] for the case of homogeneous solids). But in general, it will be necessary to write down explicitly the value of all the $\eta_i^{A_n(m)}$ in order to have the stress singularities. This study also reveals the presence of a term with Dirac delta function $\delta(x_i)$ in $\sigma_{13}^{(0x,m)}$ (30) on the interface at $x_2 = 0$. The associated coefficient is proportional to shear modulus $\mu_m$; hence it changes with $m=1$ and 2. No alternative has been found to this value. Assume now that the interface with the associated dislocation have a more general form $f(1)$ in the $x_2x_3$ - plane. Writing $B_n = \xi_n \sin \kappa_n x_3 + \delta_n \cos \kappa_n x_3$ and assuming $\xi$ small, the elastic fields $\bar{u}^{(m)}$ and $(\sigma)^{(m)}$ in the bi-material are simply (to linear terms in the amplitudes)

$$
\bar{u}^{(m)} = \bar{u}_{h}^{(0x,m)} + \bar{u}_{\xi}^{(m)},
$$

$$
(\sigma)^{(m)} = (\sigma)_{h}^{(0x,m)} + (\sigma)_{\xi}^{(m)},
$$

(45)

where

$$
\bar{u}_{\xi}^{(m)} = \sum_n \bar{u}_n^{B_n(m)},
$$

$$
(\sigma)_{\xi}^{(m)} = \sum_n (\sigma)_{B_n(m)}.
$$

(46)

Here, $\bar{u}_{h}^{(0x,m)}$ and $(\sigma)_{h}^{(0x,m)}$ are the fields of an interface straight dislocation, displaced by $x_2 = h$ from the origin. In the various elastic field expressions obtained with sinusoidal dislocations with shape $A_n$, we just have to replace $A_n$ with $B_n$ and add the symbol $\sum$ in front of the oscillating elastic fields; in addition, $x_2$ has to be replaced by $x_2 - h$. We shall proceed further by providing expressions for the elastic fields of interface climb-type edge sinusoidal dislocation ($\vec{b}_{II} = (b,0,0)$), crack tip stresses and crack extension force. These will be the subject of part II of the work.

REFERENCES

Our purpose here is to write down the differences $\Delta \tilde{u}^{\infty}$ and $(\Delta \sigma)^{\infty}$ (4) on crossing the interface at arbitrary point $P_5(x_1, x_2 = \xi, x_3)$. We use the notation $x_2 = \xi$ ($\xi = \xi_n \sin \kappa_n x_3$ small) and take the MacLaurin series expansions of the elastic fields up to terms of first order with respect to $\xi$; this means that

$$
\Delta \tilde{u}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta \tilde{u}^{\infty}(x_1, 0, x_3) + \frac{\partial \Delta \tilde{u}^{\infty}}{\partial x_2}(x_1, 0, x_3) \xi,
$$

$$
(\Delta \sigma)^{\infty}(x_1, x_2 = \xi, x_3) = (\Delta \sigma)^{\infty}(x_1, 0, x_3) + \frac{\partial (\Delta \sigma)^{\infty}}{\partial x_2}(x_1, 0, x_3) \xi. \tag{A.1}
$$

$\tilde{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$ are taken from our previous works [5, 6]. We obtain $u_i$ is the $i$-component of vector $\tilde{u}$ and $\sigma_{ij}$ the $ij$-element of the stress matrix $(\sigma)$; $i, j= 1$ to 3

$$
\Delta u_i^{\infty}(x_1, x_2 = \xi, x_3) = \Delta u_i^{(0)\infty} + \Delta u_i^{A_i\infty}
$$

$$
\Delta \sigma_{ij}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta \sigma_{ij}^{(0)\infty} + \Delta \sigma_{ij}^{A_{ij}\infty}
$$

as

$$
\Delta u_1^{(0)\infty} = 0.
$$

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\[ \Delta u_1^{(0)\infty} = \frac{\partial A_n}{\partial x_3} \frac{b C_v}{8\pi} \frac{k_1^2}{8\pi} \int_{-\infty}^\infty \frac{k_1}{(k_1^2 + \kappa_n^2)^{3/2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta u_2^{(0)\infty} = 0, \]

\[ \Delta u_3^{(0)\infty} = \frac{\partial A_n}{\partial x_3} \frac{b C_v}{8\pi} \frac{ik_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta u_3^{(0)\infty} = 0, \]

\[ \Delta u_3^{(0)\infty} = \kappa_n^2 A_n \frac{b C_v}{8\pi} \frac{ik_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta \sigma_{11}^{(0)\infty} = 0, \]

\[ \Delta \sigma_{11}^{(0)\infty} = 2Q_b \frac{\partial A_n}{\partial x_3} \frac{k_1^2 + 2\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta \sigma_{22}^{(0)\infty} = 0, \]

\[ \Delta \sigma_{22}^{(0)\infty} = 2(2Q_x - Q_b) \frac{\partial A_n}{\partial x_3} \frac{k_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta \sigma_{33}^{(0)\infty} = 0, \]

\[ \Delta \sigma_{33}^{(0)\infty} = 2Q_b \frac{\partial A_n}{\partial x_3} \frac{k_1^2 (2k_1^2 + \kappa_n^2)}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta \sigma_{12}^{(0)\infty} = 0, \]

\[ \Delta \sigma_{12}^{(0)\infty} = 2i \frac{\partial A_n}{\partial x_3} \frac{Q_b \kappa_n^2 k_1^2 + (Q_x - 2Q_b)(k_1^2 + \kappa_n^2)}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]

\[ \Delta \sigma_{13}^{(0)\infty} = 2i(Q_x - Q_b) \frac{|k_1| e^{ik_1 \xi}}{\sqrt{k_1^2 + \kappa_n^2}} \, dk_1; \]

\[ \Delta \sigma_{13}^{(0)\infty} = -2iA_n \frac{Q_b \kappa_n^2 k_1^2 + (Q_x - Q_b)(k_1^2 + \kappa_n^2)^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 \xi} \, dk_1; \]
\[
\Delta \sigma^{(0, \infty)}_{23} = 2(Q_a - Q_b) \int_{-\infty}^{\infty} \text{sgn}(k_i) e^{ik_1 x_1} dk_1, \\
\Delta \sigma^{A, \infty}_{23} = 2A_n \int_{-\infty}^{\infty} \left[ (Q_b - Q_a)k_1 + (2Q_a - Q_b)k_n^2 \right] e^{ik_1 x_1} dk_1 \xi; \\
\]
where
\[
C_v = [1/(1 - \nu_1) - 1/(1 - \nu_2)], \quad Q_b = i(C_2 - C_1)/4, \quad Q_c = i(\nu_2C_2 - \nu_1C_1)/4, \quad C_m = b\mu_m/2\pi(1 - \nu_m);
\]
\[
\text{sgn}(k_1) = k_i / |k_i|; \quad \mu_m \text{ and } \nu_m \text{ are shear modulus and Poisson’s ratio.}
\]

**APPENDIX B : PARTIAL OSCILLATING ELASTIC FIELDS (SCREW)**

The couple \((\bar{\sigma}^{A, (m)}_{3a}, \bar{\beta}^{A, (m)}_{3a})\) is obtained from \((13 a, b, d \text{ and } e)\) associated with the displacement. We have at position \(\bar{x} = (x_1, x_2, x_3)\) \(\bar{u}^{A, (m)V} = \bar{u}^{A, (m)V}_a\), \((\sigma)^{A, (m)V} = (\sigma)^{A, (m)V}_a\):
\[
u^{A, (m)V}_a = \frac{s^{(m)}_{1a}}{2\mu_m} \left( \partial_1 + \partial_2 \right) + \frac{s^{(m)}_{3a}}{2\mu_m} \left( \partial_3 \right) + A_n \left( 2K_0[\kappa_n r] \partial_1 + (1)^{-m-1} \frac{\partial \Pi_1}{\partial x_1} \right) \\
\sigma^{A, (m)V}_{1ia} = \frac{\partial A_n}{\partial x_3} \kappa_n s^{(m)}_{1a} \left( -\partial_1 + \partial_3 \right) \frac{2x_1 K_1[\kappa_n r]}{r} + \kappa_n(-\partial_1 + \partial_3) \frac{\partial I_{1a}}{\partial x_1} \right), \\
\sigma^{A, (m)V}_{12a} = \frac{\partial A_n}{\partial x_3} \kappa_n s^{(m)}_{1a} \left( -1 \right)^m \left( \frac{2x_1 K_1[\kappa_n r]}{r} - \kappa_n \Pi_1 \right), \quad (B.1) \\
\sigma^{A, (m)V}_{j3a} = A_n \kappa_n^2 s^{(m)}_{1a} \left( \delta_{j1} \left( \kappa_n^2 I_{1a} - 2K_0[\kappa_n r] \right) + \delta_{j2} (1)^{-m} \frac{\partial \Pi_1}{\partial x_1} \right), \quad \Pi = \int_{-\infty}^{\infty} e^{-\kappa_n^2} e^{ik_1 x_1} dk_1, \\
I_{1a} = \int_{-\infty}^{\infty} e^{-\kappa_n^2} e^{ik_1 x_1} dk_1, \quad (B.1) \\
Terms in brackets \| \| \text{ are operators acting on } A_n \text{ and } I_{1a}, \text{ separately; } \Pi_1 \text{ is the}
\]

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value of $\Pi_z$ for $z=1$, $r^2 = x_1^2 + x_2^2$ and subscripts $i=1$ to 3 and $j=1$ and 2; $K_n[x]$ is the nth-order modified Bessel function usually so denoted and $\delta_{ij}$ is the Kronecker delta. We stress that the various integrations (such as in $I_{1a}$ and $\Pi_z$) performed in the present study are given for spatial positions satisfying the condition $(-1)^m x_2 < 0 \ (i.e. \ (-1)^{m-1} = \text{sgn}(x_2))$ with $m = 1$ when $x_2 > \xi_n \sin \kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin \kappa_n x_3$ (half-space 2). However, this makes no difference in the elastic fields to first order in $\xi_n$.

The pair $(\bar{A}_{1b}^{(m)}, \bar{B}_{1b}^{(m)})$ is obtained using (13 b to e) associated with the displacement. We have (using the similar notations)

$$u_{1b}^{A_{1b}^{(m)}} = \frac{\partial A_n}{\partial x_2} S_{1b}^{(m)} \left(\frac{\nu_1 + \nu_2 - 2 - 2\Omega_{1b}(1 - \rho_m)\partial\Pi_1}{1 - \Omega_{1b}} \left| \frac{\partial\Pi_1}{\partial x_2} \right| + \frac{\Omega_{1b}(4 - \nu_m - 3\rho_m)}{1 - \Omega_{1b}} \frac{\partial\Pi_{\Omega_{1b}}}{\partial x_2} + 2(1 - \rho_m)\kappa_n^2 I_{1a} \right),$$

$$u_{1b}^{A_{1b}^{(m)}} = \frac{\partial A_n}{\partial x_3} S_{1b}^{(m)} \kappa_n^2 x_2 \left(\Pi_1 - \Omega_{1b} \Pi_{\Omega_{1b}} \right);$$

$$u_{2b}^{A_{1b}^{(m)}} = \frac{\partial A_n}{\partial x_1} \Omega_{1b}(4 - \nu_m - 3\rho_m)\left(\Pi_{\Omega_{1b}} - 2[1 - \rho_m \Pi_1] \right),$$

$$u_{2b}^{A_{1b}^{(m)}} = \frac{\partial A_n}{\partial x_3} \kappa_n^2 x_2 \left(\Pi_1 - \Omega_{1b} \Pi_{\Omega_{1b}} \right);$$

$$u_{3b}^{A_{1b}^{(m)}} = A_n S_{1b}^{(m)} \left(2\kappa_n^2(\rho_m - 1)\frac{\partial I_{1a}}{\partial x_1} + \frac{4 - \nu_m - 3\rho_m}{1 - \Omega_{1b}} \right) \left(\Pi_1 - \Omega_{1b} \right),$$

$$u_{3b}^{A_{1b}^{(m)}} = A_n \kappa_n^2 x_2 \left(\Pi_1 - \Omega_{1b} \right);$$

$$\sigma_{1b}^{A_{1b}^{(m)}} = \frac{\partial A_n}{\partial x_2} S_{1b}^{(m)} \left(\frac{\partial\Pi_1}{\partial x_1} \nu_1 + \nu_2 - 2 - 2\Omega_{1b}(1 - \rho_m) \left| \frac{\partial\Pi_1}{\partial x_2} \right| + \frac{\Omega_{1b}(4 - \nu_m - 3\rho_m)}{1 - \Omega_{1b}} \frac{\partial\Pi_{\Omega_{1b}}}{\partial x_2} + 2(1 - \rho_m)\kappa_n^2 I_{1a} \right),$$

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\[
\sigma_{1_{lb}}^{A_{n}(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{lb}^{(m)} \frac{\partial}{\partial x_1} \left( \kappa_n^2 x_2 (\Pi_1 - \Omega_{lb} \Pi_{lb}) + (-1)^m 2\nu_m \frac{\partial}{\partial x_2} (\Pi_1 - \Pi_{lb}) \right);
\]
\[
\sigma_{2_{lb}}^{A_{n}(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{lb}^{(m)} \frac{\partial^2}{\partial x_1 \partial x_2} \left( (4 - \nu_m - 3\rho_m) \Pi_{lb} - 2[1 - \rho_m] \Pi_1 \right),
\]
\[
\sigma_{2_{lb}}^{A_{n}(m)B} = -\frac{\partial A_n}{\partial x_3} 2s_{lb}^{(m)} \frac{\partial}{\partial x_1} \left( (1 - \Omega_{lb}) \kappa_n^2 x_2 \Pi_{lb} + (-1)^m 2(1 - \nu_m) \frac{\partial}{\partial x_2} (\Pi_1 - \Pi_{lb}) \right);
\]
\[
\sigma_{3_{lb}}^{A_{n}(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{lb}^{(m)} \left( 2\kappa_n^2 (\rho_m - 1) \frac{\partial I_{la}}{\partial x_1} + \frac{4 - \nu_m - 3\rho_m}{1 - \Omega_{lb}} \frac{\partial^2}{\partial x_1 \partial x_2} (\Pi_1 - \Pi_{lb}) \right),
\]
\[
\sigma_{3_{lb}}^{A_{n}(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{lb}^{(m)} \kappa_n^2 \left( -\kappa_n^2 x_2 \frac{\partial}{\partial x_1} + (-1)^m 2(2 - \nu_m) \frac{\partial^2}{\partial x_1 \partial x_2} \right) (\Pi_1 - \Pi_{lb});
\]
\[
\sigma_{1_{lb}}^{A_{n}(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{lb}^{(m)} (-1)^m \left( I_0 (2 - \nu_1 - \nu_2) + 2(1 - \rho_m) \kappa_n^2 \Pi_1 - [4 - \nu_m - 3\rho_m] \Omega_{lb} \kappa_n^2 \Pi_{lb} \right),
\]
\[
\sigma_{1_{lb}}^{A_{n}(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{lb}^{(m)} \kappa_n^2 \left( 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial x_2} \right) (\Pi_1 - \Omega_{lb} \Pi_{lb});
\]
\[
\sigma_{1_{lb}}^{A_{n}(m)A} = -A_n 2s_{lb}^{(m)} \kappa_n^2 \left( \frac{\partial \Pi_1 \nu_1 + \nu_2 - 2 - 2\Omega_{lb} (1 - \rho_m)}{1 - \Omega_{lb}} \right)
\]
\[
+ \frac{\partial \Pi_{lb} \Omega_{lb} (4 - \nu_m - 3\rho_m)}{1 - \Omega_{lb}} + 2(1 - \rho_m) \kappa_n^2 I_{la} \right),
\]
\[
\sigma_{1_{lb}}^{A_{n}(m)B} = -A_n 2s_{lb}^{(m)} \kappa_n^2 \left( 2\kappa_n^2 x_2 + (-1)^{m-1} 4(1 - \nu_m) \frac{\partial}{\partial x_2} \right) (\Pi_1 - \Omega_{lb} \Pi_{lb});
\]
\[
\sigma_{2_{lb}}^{A_{n}(m)A} = A_n 2s_{lb}^{(m)} (-1)^m \kappa_n^2 \frac{\partial}{\partial x_1} \left( (4 - \nu_m - 3\rho_m) \Pi_{lb} - 2[1 - \rho_m] \Pi_1 \right),
\]
\[
\sigma_{2_{lb}}^{A_{n}(m)B} = A_n 2r_{lb}^{(m)} \kappa_n^2 \frac{\partial}{\partial x_1} \left( [1 + 2(1 - \nu_m) (1 - \Omega_{lb}) \Pi_{lb} - \Pi_1 \right)
\]

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\[ I_0 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(k_n^2 + \kappa_n^2\right)x_2^2} e^{ik_n x_1} dk_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(k_n^2 + \kappa_n^2\right)|v_1|} e^{ik_n x_1} dk_1 = \frac{2\kappa_n |x_2|}{r} K_1. \]

The couple \((\sigma_{3c}^{A_{3c}}(m), \beta_{3c}^{A_{3c}}(m))\) is obtained using \((13, j, k, n \text{ and } o)\) associated with stresses. We obtain

\[ u_{1c}^{A_{3c}(m)} = \frac{\partial A_n}{\partial \chi_3} \frac{s_{1c}^{(m)}}{\kappa_n^2} \left( v_m - (2 - v_m)Q_r \right) \frac{\partial I_0}{\partial |x_2|} \]

\[-2(1 - 2v_m)\kappa_n^2 K_0[\kappa_n r] - \kappa_n^4 I_{1a}, \]

\[ u_{1c}^{A_{3c}(m)B} = \frac{\partial A_n}{\partial \chi_3} \frac{r_{1c}}{\mu_m} x_2 \left( 1 + Q_r \right) I_0 - 2\kappa_n^2 \Pi_1; \]

\[ u_{2c}^{A_{3c}(m)B} = -\frac{\partial A_n}{\partial \chi_3} \frac{r_{1c}}{\mu_m} \left[ \frac{1}{1 - m} x_2^2 \frac{\partial}{\partial |x_2|} \right] 2 \frac{\partial \Pi_1}{\partial \kappa_n} + i(Q_r - 1)I_{1c}; \]

\[ u_{3c}^{A_{3c}(m)A} = A_n \frac{s_{1c}^{(m)}}{\mu_m} \left( -2v_m \frac{\partial K_0}{\partial \chi_1} + i(2 - v_m)(1 - Q_r) \frac{\partial I_{1c}}{\partial |x_2|} + \kappa_n^2 \frac{\partial I_{1a}}{\partial \chi_1} \right); \]

\[ u_{3c}^{A_{3c}(m)B} = A_n \frac{r_{1c}}{\mu_m} \left[ \kappa_n^2 x_2 + \frac{1}{1 - m - 4(1 - v_m)} \frac{\partial}{\partial |x_2|} \right] 2 \frac{\partial \Pi_1}{\partial \kappa_n} + i(Q_r - 1)I_{1c}; \]

\[ \sigma_{1c}^{A_{3c}(m)A} = \frac{\partial A_n}{\partial \chi_3} 2s_{1c}^{(m)} \frac{1}{\kappa_n^2} \left( \frac{\partial^2 I_0}{\partial \chi_1 \partial |x_2|} \right) \left( v_m + Q_r(v_m - 2) \right) - \kappa_n^4 \frac{\partial I_{1a}}{\partial \chi_1} \]

\[ -2\kappa_n^2 (1 - 2v_m) \frac{\partial K_0}{\partial \chi_1}, \]

P.N.B. ANONGBA
\[ \sigma_{1lc}^{A(m)B} = \frac{\partial A_n}{\partial x_3} 2 r_c \left( x_2 \frac{\partial}{\partial x_1} \left[ (1 + Q_r) I_0 - 2 \kappa_n^2 \Pi_1 \right] + (-1)^{m-1} 2 \nu_m \frac{\partial}{\partial x_2} \left[ 2 \frac{\partial \Pi_1}{\partial x_1} + i(Q_r - 1) I_{lc} \right] \right); \]

\[ \sigma_{2lc}^{A(m)A} = \frac{\partial A_n}{\partial x_3} 2 s_{lc}^{(m)} \frac{1}{\kappa_n^2} \left( \frac{\partial^2 I_0}{\partial x_1 \partial x_2} \right) \left( Q_r (2 - \nu_m) - \nu_m \right) - 2 \kappa_n^2 \frac{\partial K_0}{\partial x_1} + i \kappa_n^2 (2 - \nu_m)(Q_r - 1) \frac{\partial I_{lc}}{\partial x_2} \right), \]

\[ \sigma_{2lc}^{B(m)B} = -\frac{\partial A_n}{\partial x_3} 2 r_c \left( x_2 \left[ (1 + Q_r) \frac{\partial I_0}{\partial x_1} + i \kappa_n^2 (Q_r - 1) I_{lc} \right] + (-1)^m 2 (1 - \nu_m) \left[ 4 \frac{\partial K_0}{\partial x_1} + i(1 - Q_r) \frac{\partial I_{lc}}{\partial x_1} \right] \right); \]

\[ \sigma_{3lc}^{A(m)A} = \frac{\partial A_n}{\partial x_3} 2 s_{lc}^{(m)} \left( 4 (1 - \nu_m) \frac{\partial K_0}{\partial x_1} + i(2 - \nu_m)(1 - Q_r) \frac{\partial I_{lc}}{\partial x_2} + \kappa_n^2 \frac{\partial I_{1a}}{\partial x_1} \right), \]

\[ \sigma_{3lc}^{A(m)B} = \frac{\partial A_n}{\partial x_3} 2 r_c \left( x_2 \kappa_n^2 \left[ 2 \frac{\partial \Pi_1}{\partial x_1} + i(Q_r - 1) I_{lc} \right] + (-1)^m 2 (2 - \nu_m) \left[ 4 \frac{\partial K_0}{\partial x_1} + i(1 - Q_r) \frac{\partial I_{lc}}{\partial x_1} \right] \right); \]

\[ \sigma_{12c}^{A(m)A} = \frac{\partial A_n}{\partial x_3} 2 s_{lc}^{(m)} \frac{1}{\kappa_n^2} (-1)^m \left( \frac{\partial^2 I_0}{\partial x_1} \left( \nu_m + Q_r (\nu_m - 2) \right) \right) - \kappa_n^4 \Pi_1 + \kappa_n^2 \left[ (2 - \nu_m) Q_r + \nu_m - 1 \right] I_0 \right), \]

\[ \sigma_{12c}^{A(m)B} = \frac{\partial A_n}{\partial x_3} 2 r_c \left( (1 + Q_r) I_0 - 2 \kappa_n^2 \Pi_1 + (1 - Q_r) I_0 - 2 \kappa_n^2 \Pi_1 \right); \]
\[
\sigma_{13c}^{A,m} = -A_n 2s_{1c}^{(m)} \left( (v_m - (2 - v_m)Q_r) \frac{\partial I_0}{\partial x_2} \right) - 2(1 - 2v_m) \kappa_n^2 K_0 [\kappa_n r] - \kappa_n^4 I_{1a},
\]

\[
\sigma_{13c}^{A,m} = -A_n r_{1c} \left( (-1)^{m-1} 4(1 - v_m) \frac{\partial}{\partial x_2} \left[ (1 + Q_r) I_0 - 2\kappa_n^2 \Pi_1 \right] \right) + x_2 2\kappa_n^2 \left[ (1 + Q_r) I_0 - 2\kappa_n^2 \Pi_1 \right];
\]

\[
\sigma_{23c}^{A,m} = A_n 2s_{1c}^{(m)} (-1)^m \left[ (2 - v_m)Q_r - v_m \right] \frac{\partial I_0}{\partial x_1} + i\kappa_n^2 (2 - v_m)(Q_r - 1) I_{1c} + \kappa_n^2 \frac{\partial \Pi_1}{\partial x_1},
\]

\[
\sigma_{23c}^{A,m} = A_n r_{1c} \left( i2\kappa_n^2 (3 - 2v_m)(Q_r - 1) I_{1c} + 4\kappa_n^2 \frac{\partial \Pi_1}{\partial x_1} \right) + 4(1 - v_m)(1 + Q_r) \frac{\partial I_0}{\partial x_1} + x_2 2\kappa_n^2 (-1)^m \left[ 4 \frac{\partial K_0}{\partial x_1} + i(1 - Q_r) \frac{\partial I_{1c}}{\partial x_2} \right]; \quad (B.3)
\]

\[
I_{1c} = \int_{-\infty}^{\infty} e^{-\sqrt{\kappa_n^2 + \kappa_n^2} |x_2|} dI_0.
\]

The pair \((\alpha_{3d}^{A,m}, \beta_{3d}^{A,m})\) is calculated from (13 f to i) associated with stresses. We have

\[
u_{1d}^{A,m} = \frac{\partial A_n}{\partial x_3} \frac{s_{1d}^{(m)}}{\mu_m} \left[ 2K_0 \left[ s_{2d}^{(m)} - \frac{s_{m}^{(m)} s_{3d}^{(m)}}{(1 + s_m)^2} + \frac{r_m s_{4d}^{(m)}}{(1 + r_m)^2} \right] \right]
\]

\[
- \kappa_n^2 \left[ s_{2d}^{(m)} - \frac{s_{m}^{(m)} s_{4d}^{(m)}}{1 + s_m} + \frac{r_m s_{3d}^{(m)}}{1 + r_m} \right] I_{1a} - \frac{s_m^{(m)} \frac{\partial \Pi_{-s_{m}}}{\partial x_2}}{(1 + s_m)^2} + \frac{r_m s_{4d}^{(m)}}{(1 + r_m)^2} \frac{\partial \Pi_{-r_{m}}}{\partial x_2},
\]

\[
u_{1d}^{A,m} = \frac{\partial A_n}{\partial x_3} \frac{r_{1d}^{(m)}}{\mu_m} x_2 \left( (1 + r_m) I_0 + r_m \kappa_n^2 (r_m + \tilde{Q}_r) \Pi_{-(r_m)} - \kappa_n^2 (1 - \tilde{Q}_r) \Pi_1 \right);
\]

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\[ u_{2d}^{A,(m)A} = \frac{\partial A_n}{\partial x_3} \frac{S^{(m)}_{1d}}{\mu_m} (1)^{m-1} \frac{\partial}{\partial x_1} \left[ \left( s^{(m)}_{2d} - \frac{s^{(m)}_{3d}}{1 + s_m} + \frac{s^{(m)}_{4d}}{1 + r_m} \right) \Pi_1 + \frac{s^{(m)}_{3d}}{1 + s_m} \Pi_{(-s_m)} - \frac{s^{(m)}_{4d}}{1 + r_m} \Pi_{(-r_m)} \right] \],

\[ u_{2d}^{A,(m)B} = -\frac{\partial A_n}{\partial x_3} \frac{r^{(m)}_{1d}}{\mu_m} \left[ \frac{\partial}{\partial x_1} + (-1)^{m-1} x_2 \frac{\partial^2}{\partial x_1 \partial x_2} \right] \left( r_m + \tilde{Q}_r \right) \Pi_{(-r_m)} + (1 - \tilde{Q}_r) \Pi_1 \];

\[ u_{3d}^{A,(m)A} = A_n \frac{S^{(m)}_{1d}}{\mu_m} \frac{\partial}{\partial x_1} \left[ 2K_0 \left( \frac{s^{(m)}_{4d}}{1 + r_m} - \frac{s^{(m)}_{3d}}{1 + s_m} \right) \frac{\partial^2}{\partial x_1 \partial x_2} \left( r_m + \tilde{Q}_r \right) \Pi_{(-r_m)} + (1 - \tilde{Q}_r) \Pi_1 \right] \];

\[ u_{3d}^{A,(m)B} = A_n \frac{r^{(m)}_{1d}}{\mu_m} \left[ x_2 \kappa_n^2 \frac{\partial}{\partial x_1} + (-1)^{m-1} 4 \left( 1 - v_m \right) \frac{\partial^2}{\partial x_1 \partial x_2} \right] \times \left( r_m + \tilde{Q}_r \right) \Pi_{(-r_m)} + (1 - \tilde{Q}_r) \Pi_1 \];

\[ \sigma_{11d}^{A,(m)A} = \frac{\partial A_n}{\partial x_3} \frac{2s^{(m)}_{1d}}{x_2} \frac{\partial}{\partial x_1} \left[ 2K_0 \left( \frac{s^{(m)}_{4d}}{1 + r_m} - \frac{s^{(m)}_{3d}}{1 + s_m} \right) \frac{\partial^2}{\partial x_1 \partial x_2} \right] \]

\[ \sigma_{11d}^{A,(m)B} = \frac{\partial A_n}{\partial x_3} \frac{2r^{(m)}_{1d}}{x_2} \frac{\partial}{\partial x_1} \left[ x_2 \left( 1 + r_m \right) I_0 + r_m \kappa_n^2 \left( r_m + \tilde{Q}_r \right) \Pi_{(-r_m)} - \kappa_n^2 \left( 1 - \tilde{Q}_r \right) \Pi_1 \right] \]

\[ \sigma_{22d}^{A,(m)A} = -\frac{\partial A_n}{\partial x_3} \frac{2s^{(m)}_{1d}}{x_2} \frac{\partial}{\partial x_1} \left[ 2 \left( \frac{s^{(m)}_{4d}}{1 + r_m} - \frac{s^{(m)}_{3d}}{1 + s_m} \right) K_0 \right] \]

\[ \sigma_{22d}^{A,(m)B} = -\frac{\partial A_n}{\partial x_3} \frac{2r^{(m)}_{1d}}{x_2} \left[ x_2 \left( 1 + r_m \right) \frac{\partial}{\partial x_1} \left( I_0 + \kappa_n^2 \left( r_m + \tilde{Q}_r \right) \Pi_{(-r_m)} \right) \right] \]

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\[
+ (-1)^{m-1} 2(1 - \nu_m) \frac{\partial^2}{\partial x_1 \partial x_2} \left((r_m + \tilde{Q}_r) \Pi_{(r_m)} + (1 - \tilde{Q}_r) \Pi_1 \right) \];

\[
\sigma_{33d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1d}^{(m)} \frac{\partial}{\partial x_1} \left( 2K_0 \left( \frac{s_{4d}^{(m)}}{1 + r_m^2} - \frac{s_{3d}^{(m)}}{(1 + s_m)^2} \right) + \kappa_n^2 \left[ \frac{s_{2d}^{(m)}}{1 + s_m} + \frac{s_{4d}^{(m)}}{1 + r_m} \right] \right) \frac{\partial \Pi_{(r_m)}}{\partial x_2} + \frac{s_{3d}^{(m)}}{(1 + r_m)^2} \frac{\partial \Pi_{(-r_m)}}{\partial x_2} \right) ,
\]

\[
\sigma_{33d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1d}^{(m)} \left( x_2 \kappa_n^2 \frac{\partial}{\partial x_1} + (-1)^{m-1} 2(2 - \nu_m) \frac{\partial^2}{\partial x_1 \partial x_2} \right) \frac{\partial \Pi_{(r_m)}}{\partial x_2} + \frac{s_{3d}^{(m)}}{(1 + r_m)^2} \frac{\partial \Pi_{(-r_m)}}{\partial x_2} \right) ,
\]

\[
\sigma_{12d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1d}^{(m)} (-1)^m \left( s_{2d}^{(m)} I_0 - \kappa_n^2 \left[ s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1 + s_m} + \frac{s_{4d}^{(m)}}{1 + r_m} \right] \right) \Pi_1 + \frac{s_m \kappa_n^2 s_{3d}^{(m)}}{1 + s_m} \Pi_{(-r_m)} - \frac{r_m \kappa_n^2 s_{3d}^{(m)}}{1 + r_m} \Pi_{(-r_m)} \right) ,
\]

\[
\sigma_{12d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1d}^{(m)} \left( x_2 (-1)^{m-1} \frac{\partial}{\partial x_2} \right) \frac{\partial \Pi_{(r_m)}}{\partial x_2} + \frac{s_m \kappa_n^2 s_{3d}^{(m)}}{1 + s_m} \Pi_{(-r_m)} - \frac{r_m \kappa_n^2 s_{3d}^{(m)}}{1 + r_m} \Pi_{(-r_m)} \right) ,
\]

\[
\sigma_{13d}^{A_n(m)A} = -A_n 2s_{1d}^{(m)} \kappa_n^2 \left( 2K_0 \left( \frac{s_{4d}^{(m)}}{1 + s_m^2} + \frac{r_m s_{4d}^{(m)}}{(1 + r_m)^2} \right) - \kappa_n^2 \left[ s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1 + s_m} + \frac{s_{4d}^{(m)}}{1 + r_m} \right] \right) \frac{\partial \Pi_{(-r_m)}}{\partial x_2} ,
\]

\[
\sigma_{13d}^{A_n(m)B} = -A_n 2r_{1d}^{(m)} \left( x_2 \kappa_n^2 + 2(1 - \nu_m) (-1)^{m-1} \frac{\partial}{\partial x_2} \right) \frac{\partial \Pi_{(-r_m)}}{\partial x_2} \right) \frac{\partial \Pi_{(-r_m)}}{\partial x_2} \right) ,
\]

\[
\sigma_{23d}^{A_n(m)A} = A_n 2s_{1d}^{(m)} \kappa_n^2 (-1)^m \frac{\partial}{\partial x_1} \left( \left[ s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1 + s_m} + \frac{s_{4d}^{(m)}}{1 + r_m} \right] \right) \Pi_1 .
\]

P.N.B. ANONGBA
\[
\sigma^{A_{1}(m)B}_{23d} = A_n 2s_{1d}^{(m)} \frac{\partial}{\partial x_1} \left\{ 2(1 - \nu_m)(1 + r_m) \left[ I_0 + \kappa_n^2 (r_m + \widetilde{Q}_r) \Pi_{(-s_m)} \right] + \kappa_n \left[ 1 + x_2 (-1)^{m-1} \frac{\partial}{\partial x_2} \right] \left[ (r_m + \widetilde{Q}_r) \Pi_{(-s_m)} + (1 - \widetilde{Q}_r) \Pi_1 \right] \right\}; \quad (B.4)
\]

\[
\Pi_{(-z)} \equiv \int_{-\infty}^{\infty} e^{-\frac{k_2^2 + \kappa_n^2 x_2^2}{2k_1}} e^{ik_2 x_1} dk_1.
\]

The couple \((\sigma^{A_{1}(m)}_{3e}, \beta^{A_{1}(m)}_{3e})\) is obtained from \((13 \, l, \, m, \, p \, \text{and} \, o)\) associated with stresses. We have

\[
u_{le}^{A_{1}(m)A} = \frac{\partial A_n}{\partial x_3} \frac{s_{le}^{(m)}}{\mu_m} \left( 2s_{2e}^{(m)} K_0 + \kappa_n^2 \left[ s_{3e}^{(m)} - s_{2e}^{(m)} \right] I_{1a} \right),
\]

\[
u_{le}^{A_{1}(m)B} = \frac{\partial A_n}{\partial x_3} \frac{r_{le}^{(m)}}{\mu_m} x_2 \left( I_0 - 2 \kappa_n^2 \Pi_1 \right);
\]

\[
u_{2e}^{A_{1}(m)A} = \frac{\partial A_n}{\partial x_3} \frac{s_{2e}^{(m)}}{\mu_m} \left( -1 \right)^{m-1} \left( i s_{3e}^{(m)} I_{1c} - \left[ s_{3e}^{(m)} - s_{2e}^{(m)} \right] \frac{\partial \Pi_1}{\partial x_1} \right),
\]

\[
u_{2e}^{A_{1}(m)B} = \frac{\partial A_n}{\partial x_3} \frac{r_{2e}^{(m)}}{\mu_m} \left[ 1 + x_2 (-1)^{m-1} \frac{\partial}{\partial x_2} \right] \left[ i I_{1c} - 2 \frac{\partial \Pi_1}{\partial x_1} \right];
\]

\[
u_{3e}^{A_{1}(m)A} = \frac{A_n}{\mu_m} s_{3e}^{(m)} \left( s_{3e}^{(m)} \frac{\partial}{\partial x_2} \left[ \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right] + \kappa_n^2 \left[ s_{2e}^{(m)} - s_{3e}^{(m)} \right] \frac{\partial I_{1a}}{\partial x_1} \right),
\]

\[
u_{3e}^{A_{1}(m)B} = \frac{A_n}{\mu_m} r_{3e}^{(m)} x_2 \kappa_n^2 + 4(1 - \nu_m) (-1)^{m-1} \frac{\partial}{\partial x_2} \left[ 2 \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right];
\]

\[
\sigma^{A_{1}(m)A}_{1le} = \frac{\partial A_n}{\partial x_3} 2s_{1e}^{(m)} \frac{\partial}{\partial x_1} \left( 2s_{2e}^{(m)} K_0 + \kappa_n^2 \left[ s_{3e}^{(m)} - s_{2e}^{(m)} \right] I_{1a} \right),
\]

\[
\sigma^{A_{1}(m)B}_{1le} = \frac{\partial A_n}{\partial x_3} 2r_{1e}^{(m)} \left[ x_2 \frac{\partial}{\partial x_1} \left[ I_0 - 2 \kappa_n^2 \Pi_1 \right] + (-1)^{m-1} 2 \nu_m \frac{\partial}{\partial x_2} \left[ 2 \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right] \right];
\]

P.N.B. ANONGBA
\[ \sigma_{22e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ 2s_{1e}^{(m)} \frac{\partial I_{1c}}{\partial x_2} + 2s_{2e}^{(m)} - s_{2e}^{(m)} \right] \frac{\partial K_0}{\partial x_1}, \]

\[ \sigma_{22e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n I_{1c} \right] + (-1)^m 2(1 - \nu_m) \frac{\partial}{\partial x_2} \left[ 2 \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right]; \]

\[ \sigma_{33e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ s_{3e}^{(m)} \frac{\partial}{\partial x_2} \left[ \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right] + \kappa_n^2 s_{2e}^{(m)} - s_{2e}^{(m)} \right] \frac{\partial I_{1e}}{\partial x_1}, \]

\[ \sigma_{33e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n I_{1c} \right] + (1 + 2) (-1)^{m-1} \frac{\partial}{\partial x_2} \left[ 2 \frac{\partial \Pi_1}{\partial x_1} - i I_{1c} \right]; \]

\[ \sigma_{12e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]

\[ \sigma_{12e}^{A(n)m} = \frac{\partial A_n}{\partial x_3} 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]

\[ \sigma_{13e}^{A(n)m} = -A_n 2s_{1e}^{(m)} \frac{\partial}{\partial x_2} \left[ 2s_{2e}^{(m)} - s_{2e}^{(m)} \right] \frac{\partial I_{1e}}{\partial x_1}, \]

\[ \sigma_{13e}^{A(n)m} = -A_n 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]

\[ \sigma_{23e}^{A(n)m} = A_n 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]

\[ \sigma_{23e}^{A(n)m} = A_n 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]

\[ \sigma_{23e}^{A(n)m} = A_n 2 \left[ \frac{\partial I_0}{\partial x_1} + i \kappa_n s_{3e}^{(m)} \right] \Pi_1, \]