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A STUDY OF THE ELASTIC FIELDS OF INTERFACIAL EDGE DISLOCATIONS STRAIGHT AND SINUSOIDAL USING GALERKIN VECTORS WITH THREE-DIMENSIONAL BIHARMONIC FUNCTIONS IN FOURIER FORMS (COMPLETED)

P. N. B. ANONGBA

UFR Sciences des Structures de la Matière et de Technologie, Université FHB de Cocody, 22 BP 582 Abidjan 22, Côte d'Ivoire

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*Correspondance, e-mail : *anongba@yahoo.fr*

ABSTRACT

In this study, we consider two elastic solids ($S1$) and ($S2$), of infinite sizes, welded along a non-planar surface S in the form of a corrugated sheet; more specifically, with respect to a Cartesian coordinate system x_i, the interface has the same sinusoidal shape $\xi = \xi_n \sin \kappa_n x_3$ in the $x_2 x_3$ – planes and is rectilinear in the $x_1 x_2$ – planes. We investigate the elastic fields (displacement and stress) due to a dislocation lying on that interface at the origin and running indefinitely along the x_3 – direction. The approach used is to treat the elastic fields as the difference of two quantities: 1) the first corresponds to the elastic fields of a sinusoidal dislocation at the origin in an infinitely extended homogeneous medium and 2) the second satisfies the equilibrium equations with a discontinuity, when crossing the interface, identical to that given by the elastic fields of the sinusoidal dislocation from the change in the elastic constants on the passage from ($S2$) to ($S1$). This second quantity is set using Galerkin vectors whose components are expressed in the form of Fourier series and integrals. Then equations are written that reflect the continuity of the elastic fields at the crossing of the interface. These interface boundary conditions split into two distinct groups: those corresponding to a planar interface with a straight edge dislocation at the origin and those (in the linear approximation with respect to ξ_n, assuming ξ_n to be small) proportional to the sinusoid or its spatial derivative with respect to x_3. The displacement and stress fields, provided by our analysis, for an interface straight edge dislocation, reflect the presence of the Dirac delta function in the shear stresses on the interface; a comparison is made of these findings with those previously published on the same subject.

Among the additional oscillating stresses, σ_{23} is the only one that possesses a singularity of the $1/x_1$ type on the interface. Consequently, this stress contributes a non-zero value to the crack extension force when a non-planar interface crack, with an oscillating front perpendicular to the x_1- direction, is loaded in tension.

Keywords : *linear elasticity, interface dislocations, Galerkin vector, three-dimensional biharmonic functions , Fourier forms, linear systems of equations.*

RÉSUMÉ

Une étude des champs élastiques de dislocations coins droites et sinusoïdales utilisant des vecteurs de Galerkin avec des composantes tridimensionnelles biharmoniques dans la forme de Fourier (complétée)

Dans la présente étude, on considère deux solides élastiques (*S*1) et (*S*2), de tailles infinies, soudés suivant une surface non plane ayant la forme d'une tôle ondulée; plus précisément, par rapport à un système de coordonnées cartésien x_i, l'interface a une forme sinusoïdale identique $\xi = \xi_n \sin \kappa_n x_3$ dans les plans $x_2 x_3$ et rectiligne dans les plans $x_1 x_2$. On étudie les champs élastiques (déplacement et contrainte) d'une dislocation couchée sur cette interface à l'origine et courant indéfiniment dans la direction x_3. La démarche utilisée est de considérer les champs élastiques comme la différence de deux grandeurs : 1) la première correspond aux champs élastiques d'une dislocation sinusoïdale dans un milieu homogène infiniment étendu et 2) la seconde satisfait aux équations d'équilibre avec une discontinuité, à la traversée de l'interface, identique à celle mesurée dans les expressions des champs élastiques de la dislocation sinusoïdale et qui résulte du changement des constantes élastiques au passage de l'interface, du solide (*S*2) vers le solide (*S*1). Cette deuxième quantité est définie à l'aide de vecteurs de Galerkin dont les composantes sont développées dans la forme de Fourier. On pose ensuite des équations traduisant la continuité des champs élastiques à la traversée de l'interface. Ces conditions aux bords pour l'interface se répartissent en deux groupes distincts : 1) celles qui correspondent à une interface plane avec une dislocation coin droite à l'origine et 2) celles qui (dans l'approximation linéaire par rapport à ξ_n, supposé petit) sont proportionnelles à la sinusoïde ou à sa dérivée spatiale par rapport à x_3. Les champs élastiques d'une dislocation d'interface coin droite, obtenus par l'analyse, rendent compte de la présence de la fonction delta de Dirac dans les contraintes de cisaillement sur l'interface; une comparaison est

faite de nos résultats avec ceux publiés antérieurement sur le même sujet. Parmi les contraintes oscillantes additionnelles, seule σ_{23} possède une singularité de type $1/x_1$ sur l'interface. En conséquence, cette contrainte contribue à la force d'extension de la fissure quand une fissure d'interface non plane, avec un front oscillant perpendiculaire à la direction x_1, est sollicitée en tension.

Mots-clés : *élasticité linéaire, dislocations d'interface, Vecteur de Galerkin, fonctions biharmoniques à trois dimensions, expansions en séries de Fourier, systèmes d'équations linéaires.*

I - INTRODUCTION

Consider a pair of different solids *S*1 and *S*2, of infinite sizes, welded along a non-planar sinusoidal surface *S* defined by the running point $P_S\,(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$, in such a way that *S*1 and *S*2 occupy the regions $x_2 > \xi_n \sin \kappa_n x_3$ and $x_2 < \xi_n \sin \kappa_n x_3$, respectively. The situation is shown in the ***Figure 1*** where *S*1 and *S*2 are confined for illustration purpose in a parallelepiped of finite sizes.

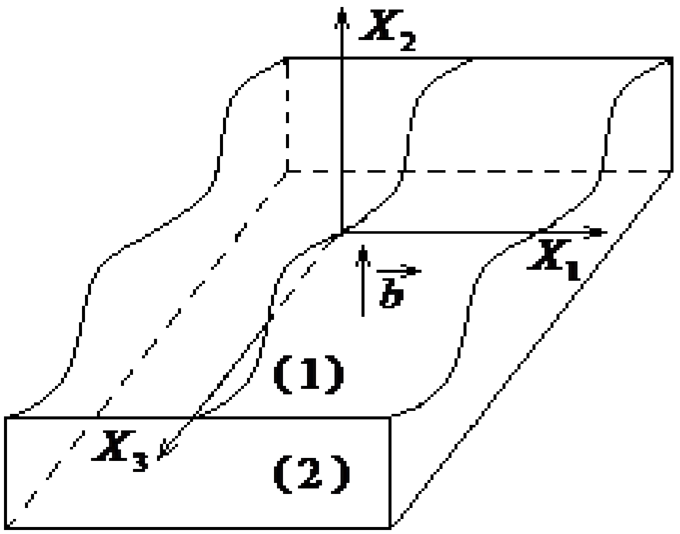

Figure 1 : *Two elastic mediums (1) and (2) welded along a non-planar sinusoidal surface and containing an interface sinusoidal dislocation at the*

origin. The dislocation lies in the Ox_2x_3 – plane and runs indefinitely in the x_3 – direction.

The present study aims at providing expressions for the displacement and stress fields of sinusoidal dislocations, lying on that interface at the origin, extending indefinitely in the x_3 – direction and spreading in the Ox_2x_3 – plane in the sinusoidal form $A_n = \xi_n \sin \kappa_n x_3$. The dislocation is edge on average for Burgers vectors in the x_1 and x_2 directions; for the former direction, the Burgers vector is perpendicular to the plane of location of the dislocation, hence this is a climb-type edge dislocation; for the latter, the Burgers vector is in the plane of the dislocation, consequently, this is a glide-type edge dislocation. In the present report, we restrict ourselves to a Burgers vector $\vec{b} = (0, b, 0)$ in the x_2 – direction. Using the results of such a study is at several levels: 1) the elastic fields due to an arbitrary form of interface dislocation in x_2x_3 planes with the same Burgers vector can be derived by superposition (Fourier series expansion); 2) a non-planar large interface crack loaded in tension in the x_2 – direction and propagating in the x_1 – direction may be represented mathematically by a continuous array of long non-straight dislocations with infinitesimal Burgers vectors $\vec{b} = (0, b, 0)$. Denote by $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ ($m = 1 \text{ and } 2$) the displacement and stress fields in the solid (*m*) due to the interface sinusoidal dislocation. We assume that the following description applies:

- $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ are continuous at the crossing of the interface

$$\vec{u}^{(1)}(P_S) = \vec{u}^{(2)}(P_S) \text{ and } (\sigma)^{(1)}(P_S) = (\sigma)^{(2)}(P_S) \quad (1)$$

- Far from the dislocation and the interface, the elastic fields in the medium (*m*) correspond to those of a sinusoidal dislocation in an infinitely extended homogeneous solid with the equal elastic constants, that we denote by $\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$, hence

$$\begin{aligned} &\vec{u}^{(m)} \to \vec{u}^{(m)\infty} \\ &(\sigma)^{(m)} \to (\sigma)^{(m)\infty} \end{aligned} \quad (2)$$

  when one moves far away in the x_2 – direction.

- The elastic fields may be expressed in the form

$$\vec{u}^{(m)} = \vec{u}^{(m)\infty} - \vec{u}^{(m)W}$$
$$(\sigma)^{(m)} = (\sigma)^{(m)\infty} - (\sigma)^{(m)W} \tag{3}$$

where $\vec{u}^{(m)W}$ *and* $(\sigma)^{(m)W}$ *satisfy the equations of equilibrium and posses the properties that follow.*

- $$\Delta\vec{u}^{\infty}(P_S) \equiv \vec{u}^{(2)\infty} - \vec{u}^{(1)\infty} = \vec{u}^{(2)W} - \vec{u}^{(1)W} \equiv \Delta\vec{u}^{W}(P_S)$$
  $$(\Delta\sigma)^{\infty}(P_S) \equiv (\sigma)^{(2)\infty} - (\sigma)^{(1)\infty} = (\sigma)^{(2)W} - (\sigma)^{(1)W} \equiv (\Delta\sigma)^{W}(P_S)\,, \tag{4}$$

  these conditions ensure the continuity of the elastic fields across the interface.

- $\vec{u}^{(m)W}$ and $(\sigma)^{(m)W}$ cancel far from the dislocation and interface; this means that

  $$\vec{u}^{(m)W} \to 0$$
  $$(\sigma)^{(m)W} \to 0 \tag{5}$$

  when $|x_2| \to \infty$; this ensures the veracity of condition (2) above.

The elastic fields $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ thus obtained are expected to be valuable representations of the physical situation illustrated in the ***Figure 1***. The associated $\vec{u}^{(m)W}$ and $(\sigma)^{(m)W}$ are investigated with the help of Galerkin vectors and corresponding equations of equilibrium. The methodology in Section 2 is essentially as follows: in Section 2.1, $\Delta\vec{u}^{\infty}(P_S)$ and $(\Delta\sigma)^{\infty}(P_S)$ are expressed in a Fourier series form that involves terms with $\exp(i\vec{k}.\vec{x})$, with $\vec{k} = (k_1, k_2, k_3)$ (k_i real numbers) and $\vec{x} = (x_1, x_2, x_3)$ vector position; in Section 2.2, a Galerkin vector with components involving $\exp(i\vec{k}.\vec{x})$ is considered. The associated elastic fields also consist of terms proportional to $\exp(i\vec{k}.\vec{x})$. These are managed in the equal Fourier series form. Then equations of the type (4) can be posed. In Section 3, the search for the appropriate elastic fields $\vec{u}^{(m)W}$ and $(\sigma)^{(m)W}$ is performed. In Section 4, calculation results are discussed and confronted to previous studies. A conclusion is made in Section 5. The beginning part of this study has been reported [1].

II - METHODOLOGY

II-1. Interface boundary values carried by the elastic fields of a sinusoidal dislocation in an homogeneous solid

The elastic fields due to a sinusoidal edge dislocation ($\vec{b} = (0, b, 0)$) lying in the Ox_2x_3 – plane in the sinusoidal form $A_n(x_3) = \xi_n \sin \kappa_n x_3$ have been provided in infinite series forms by Anongba [2]. In a similar way as in our previous studies [3-7], we shall assume ξ_n small and limit the elastic solutions up to terms of first order with respect to ξ_n. In this way, the elastic fields consist of two terms:

$$\vec{u}^{(m)\infty} = \vec{u}^{(0)(m)\infty} + \vec{u}^{A_n(m)\infty}$$
$$(\sigma)^{(m)\infty} = (\sigma)^{(0)(m)\infty} + (\sigma)^{A_n(m)\infty} \tag{6}$$

where $\vec{u}^{(0)(m)\infty}$ *and* $(\sigma)^{(0)(m)\infty}$ *are of zero order with respect to* ξ_n *corresponding to the fields of a straight edge dislocation;* $\vec{u}^{A_n(m)\infty}$ *and* $(\sigma)^{A_n(m)\infty}$ *are proportional to the sinusoid* $A_n(x_3)$ *or to its spatial derivative* $\partial A_n / \partial x_3$.

Our purpose here is to write down the differences $\Delta\vec{u}^{\infty}$ and $(\Delta\sigma)^{\infty}$ (4) on crossing the interface at arbitrary point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$. We use the notation $x_2 = \xi$ ($\xi = \xi_n \sin \kappa_n x_3$ small) and take the MacLaurin series expansions of the elastic fields up to terms of first order with respect to ξ; this means that

$$\Delta\vec{u}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta\vec{u}^{\infty}(x_1, 0, x_3) + \frac{\partial \Delta\vec{u}^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi \,,$$
$$(\Delta\sigma)^{\infty}(x_1, x_2 = \xi, x_3) = (\Delta\sigma)^{\infty}(x_1, 0, x_3) + \frac{\partial (\Delta\sigma)^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi \,. \tag{7}$$

$\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$ are taken from our previous works [2, 5,6]; we obtain (u_i is the *i*-component of vector $\vec{u}$ and σ_{ij} the *ij*-element of the stress matrix (σ); *i*, *j* = 1 to 3)

$$\Delta u_i^{\infty}(x_1, x_2 = \xi, x_3) = \Delta u_i^{(0)\infty} + \Delta u_i^{A_n\infty}$$

$$\Delta \sigma_{ij}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta \sigma_{ij}^{(0)\infty} + \Delta \sigma_{ij}^{A_n\infty}$$

as

$$\Delta u_1^{(0)\infty} = \frac{bC_{\nu}}{4\pi}\ln|x_1| = -\frac{bC_{\nu}}{8\pi}\int_{-\infty}^{\infty}\frac{1}{|k_1|}e^{ik_1x_1}dk_1$$

$$\Delta u_1^{A_n\infty} = \frac{bC_{\nu}\kappa_n A_n}{4\pi|x_1|}\left(\kappa_n|x_1|K_0 + K_1\right)\xi = -\frac{bC_{\nu}A_n}{8\pi}\int_{-\infty}^{\infty}\frac{k_1^2}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1\xi$$

$$\Delta u_2^{(0)\infty} = \frac{bC_{\nu}}{4\pi x_1}\xi = -\frac{ibC_{\nu}}{8\pi}\int_{-\infty}^{\infty}\operatorname{sgn}(k_1)e^{ik_1x_1}dk_1\xi$$

$$\Delta u_2^{A_n\infty} = -\frac{bC_{\nu}\kappa_n A_n}{4\pi}\operatorname{sgn}(x_1)K_1 = \frac{ibC_{\nu}A_n}{8\pi}\int_{-\infty}^{\infty}\frac{k_1}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1$$

$$\Delta u_3^{(0)\infty} = 0$$

$$\Delta u_3^{A_n\infty} = -\frac{bC_{\nu}\kappa_n}{4\pi}\frac{\partial A_n}{\partial x_3}\operatorname{sgn}(x_1)K_1\xi = \frac{ibC_{\nu}}{8\pi}\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{k_1}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1\xi$$

$$\Delta \sigma_{11}^{(0)\infty} = \frac{C_2 - C_1}{x_1} = -2Q_b\int_{-\infty}^{\infty}\operatorname{sgn}(k_1)e^{ik_1x_1}dk_1$$

$$\Delta \sigma_{11}^{A_n\infty} = \frac{(C_2 - C_1)\kappa_n A_n}{x_1}\left(3\kappa_n K_0 + \frac{(6+\kappa_n^2x_1^2)K_1}{|x_1|}\right)\xi$$

$$= 2Q_bA_n\int_{-\infty}^{\infty}\frac{k_1(3k_1^2+2\kappa_n^2)}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1\xi$$

$$\Delta \sigma_{22}^{(0)\infty} = \frac{C_2 - C_1}{x_1} = -2Q_b\int_{-\infty}^{\infty}\operatorname{sgn}(k_1)e^{ik_1x_1}dk_1$$

$$\Delta \sigma_{22}^{A_n\infty} = -\frac{(C_2 - C_1)\kappa_n A_n}{x_1}\left(\kappa_n K_0 + \frac{2K_1}{|x_1|}\right)\xi$$

$$= -2Q_bA_n\int_{-\infty}^{\infty}k_1\sqrt{k_1^2+\kappa_n^2}\,e^{ik_1x_1}dk_1\xi$$

$$\Delta\sigma_{33}^{(0)\infty} = -\frac{8iQ_c}{x_1} = -4Q_c\int_{-\infty}^{\infty}\mathrm{sgn}(k_1)e^{ik_1x_1}dk_1$$

$$\Delta\sigma_{33}^{A_n\infty} = -\frac{\kappa_n A_n}{x_1}\left(8iQ_c\kappa_n K_0 + \left(\frac{16iQ_c}{|x_1|} + \kappa_n^2(C_2 - C_1)|x_1|\right)K_1\right)\xi$$

$$= 2A_n\int_{-\infty}^{\infty}\frac{k_1(2Q_c k_1^2 + (2Q_c + Q_b)\kappa_n^2)}{\sqrt{k_1^2 + \kappa_n^2}}e^{ik_1x_1}dk_1\xi$$

$$\Delta\sigma_{12}^{(0)\infty} = \frac{C_2 - C_1}{x_1^2}\xi = 2iQ_b\int_{-\infty}^{\infty}|k_1|e^{ik_1x_1}dk_1\xi$$

$$\Delta\sigma_{12}^{A_n\infty} = \kappa_n A_n\left(4iQ_c\kappa_n K_0 - \frac{(C_2 - C_1)K_1}{|x_1|}\right)$$

$$= -2iA_n\int_{-\infty}^{\infty}\frac{Q_b k_1^2 + (Q_b - Q_c)\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}}e^{ik_1x_1}dk_1$$

$$\Delta\sigma_{13}^{(0)\infty} = 0$$

$$\Delta\sigma_{13}^{A_n\infty} = 4i\kappa_n\frac{\partial A_n}{\partial x_3}\left(Q_b\kappa_n K_0 + \frac{(2Q_b - Q_c)K_1}{|x_1|}\right)\xi$$

$$= 2i\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{(Q_c - 2Q_b)k_1^2 + (Q_c - Q_b)\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}}e^{ik_1x_1}dk_1\xi$$

$$\Delta\sigma_{23}^{(0)\infty} = 0$$

$$\Delta\sigma_{23}^{A_n\infty} = -4iQ_c\kappa_n\frac{\partial A_n}{\partial x_3}\mathrm{sgn}(x_1)K_1 = -2Q_c\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{k_1}{\sqrt{k_1^2 + \kappa_n^2}}e^{ik_1x_1}dk_1 \qquad (8)$$

where

$C_\nu = [1/(1 - \nu_1) - 1/(1 - \nu_2)]$, $Q_b = i(C_2 - C_1)/4$,
$Q_c = i(\nu_2 C_2 - \nu_1 C_1)/4$, $C_m = b\mu_m/2\pi(1 - \nu_m)$;

K_i is the *i*th-order modified Bessel function with argument $\kappa_n|x_1|$ and $\mathrm{sgn}(k_1) = k_1/|k_1|$; μ_m and ν_m are shear modulus and Poisson's ratio. In the various expressions in (8), constant terms are omitted.

II-2. Galerkin vectors and interface boundary conditions

A Galerkin vector $\vec{V}(\vec{x})$ is a vector whose components are biharmonic spatial functions ($\Delta\Delta\,\vec{V} = 0$; Δ Laplace operator) in order to satisfy the equilibrium equations with zero body forces. Then the associated displacement $\vec{u}$ is expressed as

$$2\mu\vec{u} = 2(1-\nu)\Delta\vec{V} - \vec{\nabla}(\vec{\nabla}.\vec{V}) \tag{9}$$

where μ and ν are shear modulus and Poisson's ratio respectively; $\vec{\nabla}$ is the operator nabla, $\vec{\nabla} = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$. The stress field (σ) is obtained from the displacement $\vec{u}$ by partial differentiation with respect to coordinates x_i. The matter is treated in a number of books ([8-12], among many others). For the present problem, we arrive at Galerkin vectors with only one non-zero x_2 – component, arranged in the form

$$V_2(\vec{x}) = \overline{\alpha}_2(\vec{k})e^{i\vec{k}.\vec{x}} + \overline{\beta}_2(\vec{k})x_2 e^{i\vec{k}.\vec{x}} \tag{10}$$

under the condition $\vec{k}^2 = k_1^2 + k_2^2 + k_3^2 = 0$ that ensures the biharmonicity of V_2. For V_2 to cancel far from the interface, we write

$$k_2 = k_2^{(m)} \equiv (-1)^{m-1} i \sqrt{k_1^2 + k_3^2} \tag{11}$$

with $m = 1$ when $x_2 > \xi_n \sin \kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin \kappa_n x_3$ (half-space 2). We use the notations

$$\vec{k}^{(m)} \equiv (k_1, k_2^{(m)}, k_3),\ \overline{\alpha}_2^{(m)} \equiv \overline{\alpha}_2(\vec{k}^{(m)}),\ \overline{\beta}_2^{(m)} \equiv \overline{\beta}_2(\vec{k}^{(m)});$$

hence for half-space 1 ($x_2 > \xi_n \sin \kappa_n x_3$), solid (1)

$$V_2(\vec{x}) \equiv V_2^{(1)}(\vec{x}) = \overline{\alpha}_2^{(1)} e^{i\vec{k}^{(1)}.\vec{x}} + \overline{\beta}_2^{(1)} x_2 e^{i\vec{k}^{(1)}.\vec{x}}$$

and for half-space 2 ($x_2 < \xi_n \sin \kappa_n x_3$), solid (2)

$$V_2(\vec{x}) \equiv V_2^{(2)}(\vec{x}) = \overline{\alpha}_2^{(2)} e^{i\vec{k}^{(2)}.\vec{x}} + \overline{\beta}_2^{(2)} x_2 e^{i\vec{k}^{(2)}.\vec{x}}.$$

The elastic fields corresponding to V_2 (10) may be first calculated starting with (9); then, more general forms $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ are constructed from the previous ones by superposition over k_1 and k_3 (here the superscript V is just a notation, not to be confused with $\|\vec{V}\|$). For $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ to conform with $\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$ (6), the summation over k_1 is continuous and that over k_3 is discrete. k_3 takes three values: $-\kappa_n$, 0, κ_n. The fields corresponding to $k_3 = 0$ are denoted $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ and terms associated with $k_3 = -\kappa_n$ and κ_n are merged to form expressions denoted $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$; this is made possible by requiring that

$$\overline{\alpha}_2^{(m)}(\kappa_n) \equiv -\overline{\alpha}_2^{(m)}(-\kappa_n)\,,\ \overline{\beta}_2^{(m)}(\kappa_n) \equiv -\overline{\beta}_2^{(m)}(-\kappa_n)\,. \tag{12}$$

In (12), $\overline{\alpha}_2^{(m)}(\kappa_n)$ stands for $\overline{\alpha}_2(k_1, k_2^{(m)}, \kappa_n)$. We write

$$\begin{aligned} \vec{u}^{(m)V} &= \vec{u}^{(0)(m)V} + \vec{u}^{A_n(m)V} \\ (\sigma)^{(m)V} &= (\sigma)^{(0)(m)V} + (\sigma)^{A_n(m)V} \end{aligned}\ . \tag{13}$$

$\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ are x_3 – independent; $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$ are proportional to the sinusoid $A_n(x_3)$ or to its spatial derivative $\partial A_n / \partial x_3$. Here also, for points P_S on the interface, $\Delta\vec{u}^V$ and $(\Delta\sigma)^V$ are expanded up to terms of first order with respect to $x_2 = \xi$ in a similar manner as in (8) for $\Delta\vec{u}^\infty$ and $(\Delta\sigma)^\infty$. Requiring $\Delta\vec{u}^V = \Delta\vec{u}^\infty$ and $(\Delta\sigma)^V = (\Delta\sigma)^\infty$ lead to the following equations, writing first the conditions corresponding to $k_3 = 0$ (i.e. $\Delta u_i^{(0)V} = \Delta u_i^{(0)\infty}$ and $\Delta\sigma_{ij}^{(0)V} = \Delta\sigma_{ij}^{(0)\infty}$).

$$\Delta u_1^{(0)V} = \Delta u_1^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2} + \frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right) + \left(\frac{\overline{\beta}_2^{(2)}}{\mu_2} - \frac{\overline{\beta}_2^{(1)}}{\mu_1}\right) = -\frac{ibC_V}{4\pi}\frac{\operatorname{sgn}(k_1)}{k_1^2} \tag{a}$$

$$|k_1|\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2} - \frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right) + 2\left(\frac{\overline{\beta}_2^{(2)}}{\mu_2} + \frac{\overline{\beta}_2^{(1)}}{\mu_1}\right) = 0 \tag{b}$$

$$\Delta u_2^{(0)V} = \Delta u_2^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2} - \frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right) - 2\left(\frac{(1-2\nu_2)\overline{\beta}_2^{(2)}}{\mu_2} + \frac{(1-2\nu_1)\overline{\beta}_2^{(1)}}{\mu_1}\right) = 0 \qquad \text{(c)}$$

$$|k_1|\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2} + \frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right) - \left(\frac{(1-4\nu_2)\overline{\beta}_2^{(2)}}{\mu_2} - \frac{(1-4\nu_1)\overline{\beta}_2^{(1)}}{\mu_1}\right) = \frac{ibC_\nu}{4\pi}\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad \text{(d)}$$

$$\Delta\sigma_{11}^{(0)V} = \Delta\sigma_{11}^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\overline{\alpha}_2^{(2)} + \overline{\alpha}_2^{(1)}\right) + (1+2\nu_2)\overline{\beta}_2^{(2)} - (1+2\nu_1)\overline{\beta}_2^{(1)} = -2Q_b\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (e)$$

$$|k_1|\left(\overline{\alpha}_2^{(2)} - \overline{\alpha}_2^{(1)}\right) + 2(1+\nu_2)\overline{\beta}_2^{(2)} + 2(1+\nu_1)\overline{\beta}_2^{(1)} = 0 \qquad (f)$$

$$\Delta\sigma_{22}^{(0)V} = \Delta\sigma_{22}^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\overline{\alpha}_2^{(2)} + \overline{\alpha}_2^{(1)}\right) - (1-2\nu_2)\overline{\beta}_2^{(2)} + (1-2\nu_1)\overline{\beta}_2^{(1)} = 2Q_b\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (g)$$

$$|k_1|\left(\overline{\alpha}_2^{(2)} - \overline{\alpha}_2^{(1)}\right) + 2\nu_2\overline{\beta}_2^{(2)} + 2\nu_1\overline{\beta}_2^{(1)} = 0 \qquad (h)$$

$$\Delta\sigma_{33}^{(0)V} = \Delta\sigma_{33}^{(0)\infty} \Rightarrow$$

$$\nu_2\overline{\beta}_2^{(2)} - \nu_1\overline{\beta}_2^{(1)} = -2Q_c\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (i)$$

$$\nu_2\overline{\beta}_2^{(2)} + \nu_1\overline{\beta}_2^{(1)} = 0 \qquad (j)$$

$$\Delta\sigma_{12}^{(0)V} = \Delta\sigma_{12}^{(0)\infty} \Rightarrow (h) \text{ and } (e) \text{ above} \qquad (14)$$

In ***Equations*** (14 *a* to *j*) above, $\overline{\alpha}_2^{(m)}$ stands for $\overline{\alpha}_2(k_1, k_2^{(m)}, k_3 = 0)$ .The conditions corresponding to $\Delta u_i^{A_nV} = \Delta u_i^{A_n\infty}$ and $\Delta\sigma_{ij}^{A_nV} = \Delta\sigma_{ij}^{A_n\infty}$ are now listed as :

$$\Delta u_1^{A_nV} = \Delta u_1^{A_n\infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2} + \frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right) + \frac{\overline{\beta}_2^{(2)}}{\mu_2} - \frac{\overline{\beta}_2^{(1)}}{\mu_1} = 0 \qquad (a)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2}-\frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right)+2\left(\frac{\overline{\beta}_2^{(2)}}{\mu_2}+\frac{\overline{\beta}_2^{(1)}}{\mu_1}\right)=-\frac{bC_v\xi_n}{8\pi}\frac{k_1}{k_1^2+\kappa_n^2} \qquad (b)$$

$\Delta u_2^{A_nV}=\Delta u_2^{A_n\infty}\Rightarrow$ (c) and (d) (displayed below)

$$\sqrt{k_1^2+\kappa_n^2}\left(-\frac{\overline{\alpha}_2^{(2)}}{\mu_2}+\frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right)+2\left(\frac{(1-2\nu_2)\overline{\beta}_2^{(2)}}{\mu_2}+\frac{(1-2\nu_1)\overline{\beta}_2^{(1)}}{\mu_1}\right)=\frac{bC_v\xi_n}{8\pi}\frac{k_1}{k_1^2+\kappa_n^2}$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\frac{\overline{\alpha}_2^{(2)}}{\mu_2}+\frac{\overline{\alpha}_2^{(1)}}{\mu_1}\right)-\frac{(1-4\nu_2)\overline{\beta}_2^{(2)}}{\mu_2}+\frac{(1-4\nu_1)\overline{\beta}_2^{(1)}}{\mu_1}=0$$

$\Delta u_3^{A_nV}=\Delta u_3^{A_n\infty}\Rightarrow$ (a) and (b) above

$\Delta\sigma_{11}^{A_nV}=\Delta\sigma_{11}^{A_n\infty}\Rightarrow$

$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)+[(1+2\nu_2)k_1^2+2\nu_2\kappa_n^2]\overline{\beta}_2^{(2)}$$
$$-[(1+2\nu_1)k_1^2+2\nu_1\kappa_n^2]\overline{\beta}_2^{(1)}=0 \qquad (e)$$

$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2[(1+\nu_2)k_1^2+\nu_2\kappa_n^2]\overline{\beta}_2^{(2)}$$
$$+2[(1+\nu_1)k_1^2+\nu_1\kappa_n^2]\overline{\beta}_2^{(1)}=-\frac{iQ_b\xi_nk_1(3k_1^2+2\kappa_n^2)}{k_1^2+\kappa_n^2} \qquad (f)$$

$\Delta\sigma_{22}^{A_nV}=\Delta\sigma_{22}^{A_n\infty}\Rightarrow$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)-(1-2\nu_2)\overline{\beta}_2^{(2)}+(1-2\nu_1)\overline{\beta}_2^{(1)}=0 \qquad (g)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2(\nu_2\overline{\beta}_2^{(2)}+\nu_1\overline{\beta}_2^{(1)})=-\frac{iQ_b\xi_nk_1}{k_1^2+\kappa_n^2} \qquad (h)$$

$\Delta\sigma_{33}^{A_nV}=\Delta\sigma_{33}^{A_n\infty}\Rightarrow$

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)+[(1+2\nu_2)\kappa_n^2+2\nu_2k_1^2]\overline{\beta}_2^{(2)}$$
$$-[(1+2\nu_1)\kappa_n^2+2\nu_1k_1^2]\overline{\beta}_2^{(1)}=0 \qquad (i)$$

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2[(1+\nu_2)\kappa_n^2+\nu_2k_1^2]\overline{\beta}_2^{(2)}$$

$$+\,2[(1+\nu_1)\kappa_n^2+\nu_1 k_1^2]\overline{\beta}_2^{(1)} = -\frac{i\xi_n k_1[2Q_c k_1^2+(Q_b+2Q_c)\kappa_n^2]}{k_1^2+\kappa_n^2} \qquad (j)$$

$$\Delta\sigma_{12}^{A_n V} = \Delta\sigma_{12}^{A_n\infty} \Rightarrow$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2(\nu_2\overline{\beta}_2^{(2)}+\nu_1\overline{\beta}_2^{(1)}) = -\frac{i\xi_n[Q_b k_1^2+(Q_b-Q_c)\kappa_n^2]}{k_1(k_1^2+\kappa_n^2)} \qquad (k)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)+(1+2\nu_2)\overline{\beta}_2^{(2)}-(1+2\nu_1)\overline{\beta}_2^{(1)} = 0 \qquad (l)$$

$$\Delta\sigma_{13}^{A_n V} = \Delta\sigma_{13}^{A_n\infty} \Rightarrow$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)+\overline{\beta}_2^{(2)}-\overline{\beta}_2^{(1)} = 0 \qquad (m)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2(\overline{\beta}_2^{(2)}+\overline{\beta}_2^{(1)}) = \frac{i\xi_n[(Q_c-2Q_b)k_1^2+(Q_c-Q_b)\kappa_n^2]}{k_1(k_1^2+\kappa_n^2)} \qquad (n)$$

$$\Delta\sigma_{23}^{A_n V} = \Delta\sigma_{23}^{A_n\infty} \Rightarrow$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}-\overline{\alpha}_2^{(1)}\right)+2(\nu_2\overline{\beta}_2^{(2)}+\nu_1\overline{\beta}_2^{(1)}) = -\frac{iQ_c\xi_n k_1}{k_1^2+\kappa_n^2} \qquad (o)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_2^{(2)}+\overline{\alpha}_2^{(1)}\right)+(1+2\nu_2)\overline{\beta}_2^{(2)}-(1+2\nu_1)\overline{\beta}_2^{(1)} = 0 \qquad (p) \qquad (15)$$

Next, we are concerned with satisfying boundary conditions: (14) leads to the displacement and stress fields due to an interface straight edge dislocation ($\vec{b}=(0,b,0)$) parallel to the x_3 – direction at the origin; the interface is the Ox_1x_3 – plane. (15) provides the complementary terms (to first order in ξ_n) in the elastic fields of an interfacial sinusoidal edge dislocation.

III - CALCULATION RESULTS

III-1. Displacement and stress fields due to an interface straight edge dislocation

III-1-1. Partial elastic fields associated with interface boundary conditions

Three distinct couples of values for $(\overline{\alpha}_2^{(m)}, \overline{\beta}_2^{(m)})$ are extracted from (14); these are :

(a) $\overline{\alpha}_2^{(m)} = \dfrac{\nu_m C_m Q_a}{k_1^3} \equiv \overline{\alpha}_{2a}^{(m)}$; $\overline{\beta}_2^{(m)} = (-1)^{m-1} \dfrac{C_m Q_a}{2} \dfrac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{2a}^{(m)}$

(b) $\overline{\alpha}_2^{(m)} = \dfrac{2\nu_m Q_b}{k_1^3} \equiv \overline{\alpha}_{2b}^{(m)}$; $\overline{\beta}_2^{(m)} = (-1)^{m-1} Q_b \dfrac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{2b}^{(m)}$

(c) $\overline{\alpha}_2^{(m)} = \dfrac{\overline{V}_c}{k_1^3} \equiv \overline{\alpha}_{2c}$, independent of *m*; $\overline{\beta}_2^{(m)} = (-1)^{m-1} \dfrac{Q_c}{\nu_m} \dfrac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{2c}^{(m)}$ (16)

where

$$Q_a = i(\nu_1 - \nu_2) / [(1-2\nu_2)(1-\nu_1) + (1-2\nu_1)(1-\nu_2)] \,,$$
$$\overline{V}_c = Q_b + 2Q_c[1 - (\nu_1 + \nu_2)/4\nu_1\nu_2] \,.$$

None of these couples satisfies the entire conditions (14). For each couple, we display below the associated elastic fields $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ defined in Section 2.2. A superposition of these partial fields will provide the complete form of solution. The couple $(\overline{\alpha}_{2a}^{(m)}, \overline{\beta}_{2a}^{(m)})$ is obtained from (14 *a* to *d*) associated with the displacement. We have at position

$\vec{x} = (x_1, x_2, x_3)$ $\left(\vec{u}^{(0)(m)V} \equiv \vec{u}_a^{(0)(m)V}, (\sigma)^{(0)(m)V} \equiv (\sigma)_a^{(0)(m)V}\right)$

$$u_{1a}^{(0)(m)V} = \frac{iC_m Q_a}{2\mu_m}\left((-1)^{m-1}(1-2\nu_m)\ln|x_1|\delta_A(x_2) + \frac{x_2^2}{r^2}\right),$$

$$u_{2a}^{(0)(m)V} = \frac{ibQ_a}{2\pi}\left(-\tan^{-1}\left(\frac{x_1}{|x_2|}\right) + (-1)^m \frac{1}{2(1-\nu_m)}\frac{x_1 x_2}{r^2}\right),$$

$$\sigma_{11a}^{(0)(m)V} = iC_m Q_a\left((-1)^{m-1}\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] - 2\operatorname{sgn}(x_2)\frac{x_1 x_2^2}{r^4}\right),$$

$$\sigma_{22a}^{(0)(m)V} = iC_m Q_a\left((-1)^{m-1}\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] + 2\operatorname{sgn}(x_2)\frac{x_1 x_2^2}{r^4}\right),$$

$$\sigma_{33a}^{(0)(m)V} = (-1)^{m-1} 2i\nu_m C_m Q_a\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right],$$

$$\sigma_{12a}^{(0)(m)V} = (-1)^m iC_m Q_a \frac{x_2(x_2^2 - x_1^2)}{r^4}; \qquad (17)$$

$$\overline{J}_1 = \int_0^\infty \sin k_1 x_1 dk_1 \,.$$

Here, δ_A has the following definition: $\delta_A(x_2) = 0$ when $x_2 \neq 0$ and $\delta_A(x_2) = 1$ when $x_2 = 0$; $r^2 = x_1^2 + x_2^2$: Constant terms are omitted in the displacement. It can easily be verified that $u_{1a}^{(0)(m)} \equiv u_1^{(0)(m)\infty} - u_{1a}^{(0)(m)V}$ have the equal expression with *m* on the interface $x_2 = 0$; but the stresses $\sigma_{12a}^{(0)(m)} \equiv \sigma_{12}^{(0)(m)\infty} - \sigma_{12a}^{(0)(m)V}$ and $\sigma_{22a}^{(0)(m)}$ defined in a similar manner exhibit different factors (with *m*) that multiply the equal spatial functions $x_2(x_1^2 - x_2^2)/r^4$ and $x_1(x_1^2 + 3x_2^2)/r^4$ respectively. The pair $(\overline{\alpha}_{2b}^{(m)}, \overline{\beta}_{2b}^{(m)})$ is obtained using (14 *e* to *h*) associated with the stresses. This gives (using the similar notations)

$$u_{1b}^{(0)(m)V} = \frac{iQ_b}{\mu_m}\left((-1)^{m-1}(1-2\nu_m)\ln|x_1|\delta_A(x_2) + \frac{x_2^2}{r^2}\right),$$

$$u_{2b}^{(0)(m)V} = \frac{iQ_b}{\mu_m}\left(-2(1-\nu_m)\tan^{-1}\left(\frac{x_1}{|x_2|}\right) + (-1)^m\frac{x_1x_2}{r^2}\right),$$

$$\sigma_{11b}^{(0)(m)V} = 2iQ_b\left((-1)^{m-1}\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] - 2\,\mathrm{sgn}(x_2)\frac{x_1x_2^2}{r^4}\right),$$

$$\sigma_{22b}^{(0)(m)V} = 2iQ_b\left((-1)^{m-1}\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] + 2\,\mathrm{sgn}(x_2)\frac{x_1x_2^2}{r^4}\right),$$

$$\sigma_{33b}^{(0)(m)V} = (-1)^{m-1}4i\nu_mQ_b\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right],$$

$$\sigma_{12b}^{(0)(m)V} = (-1)^m 2iQ_b\frac{x_2(x_2^2 - x_1^2)}{r^4}. \tag{18}$$

It can be verified that $u_{1b}^{(0)(m)} \equiv u_1^{(0)(m)\infty} - u_{1b}^{(0)(m)V}$ are not identical with *m* on the interface; but the stresses $\sigma_{12b}^{(0)(m)} \equiv \sigma_{12}^{(0)(m)\infty} - \sigma_{12b}^{(0)(m)V}$ and $\sigma_{22b}^{(0)(m)}$ (similar notations) exhibit the identical expressions with *m* (= 1 and 2) with the equal factor $(C_1 + C_2)/2$ multiplying the spatial functions $x_2(x_1^2 - x_2^2)/r^4$ and $x_1(x_1^2 + 3x_2^2)/r^4$ respectively. The couple $(\overline{\alpha}_{2c}, \overline{\beta}_{2c}^{(m)})$ is calculated using (14 *g* to *j*) associated with the stresses. This leads (similar notations apply)

$$u_{1c}^{(0)(m)V} = \frac{i}{\nu_m\mu_m}\left((-1)^m(\nu_m\overline{V}_c - Q_c)\ln|x_1|\delta_A(x_2) + Q_c\frac{x_2^2}{r^2}\right),$$

$$u_{2c}^{(0)(m)V} = -\frac{i}{\nu_m\mu_m}\left([2(1-2\nu_m)Q_c + \nu_m\overline{V}_c]\tan^{-1}\left(\frac{x_1}{|x_2|}\right) + (-1)^{m-1}Q_c\frac{x_1x_2}{r^2}\right),$$

$$\sigma_{11c}^{(0)(m)V} = \frac{2i}{\nu_m}\left((-1)^{m-1}[(1+2\nu_m)Q_c - \nu_m \overline{V}_c]\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] - 2Q_c\,\mathrm{sgn}(x_2)\frac{x_1 x_2^2}{r^4}\right),$$

$$\sigma_{22c}^{(0)(m)V} = \frac{2i}{\nu_m}\left((-1)^{m-1}[(1-2\nu_m)Q_c + \nu_m \overline{V}_c]\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right] + 2Q_c\,\mathrm{sgn}(x_2)\frac{x_1 x_2^2}{r^4}\right),$$

$$\sigma_{33c}^{(0)(m)V} = (-1)^{m-1}4iQ_c\left[\frac{x_1}{r^2} + \left(\overline{J}_1 - \frac{x_1}{r^2}\right)\delta_A(x_2)\right],$$

$$\sigma_{12c}^{(0)(m)V} = \frac{2ix_2}{r^2}\left((2Q_c - \overline{V}_c)\,\mathrm{sgn}(x_2) + (-1)^m \frac{Q_c}{\nu_m}\frac{(x_2^2 - x_1^2)}{r^2}\right) + 2\pi i(2Q_c - \overline{V}_c)\delta(x_1)\delta_A(x_2) \quad (19)$$

where $\delta(x_1)$ is the well-known Dirac delta function in $\sigma_{12c}^{(0)(m)V}$ when $x_2 = 0$.

III-1-2. Boundary conditions

We define the elastic fields $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ of an interface straight edge dislocation as

$$\vec{u}^{(0)(m)} = \vec{u}^{(0)(m)\infty} - \vec{u}^{(0)(m)W}$$
$$(\sigma)^{(0)(m)} = (\sigma)^{(0)(m)\infty} - (\sigma)^{(0)(m)W} \quad (20)$$

with

$$\vec{u}^{(0)(m)W} = \eta_a^{(m)}\vec{u}_a^{(0)(m)V} + \eta_b^{(m)}\vec{u}_b^{(0)(m)V} + \eta_c^{(m)}\vec{u}_c^{(0)(m)V}\,.$$
$$(\sigma)^{(0)(m)W} = \eta_a^{(m)}(\sigma)_a^{(0)(m)V} + \eta_b^{(m)}(\sigma)_b^{(0)(m)V} + \eta_c^{(m)}(\sigma)_c^{(0)(m)V} \quad (21)$$

Again $\vec{u}^{(0)(m)\infty}$ and $(\sigma)^{(0)(m)\infty}$ are due to a straight edge dislocation ($\vec{b} = (0, b, 0)$) parallel to the x_3 – direction at the origin in an infinitely extended homogeneous medium (*m*) (see [2, 5, 6]); $\vec{u}_{a\text{ to }c}^{(0)(m)V}$ and $(\sigma)_{a\text{ to }c}^{(0)(m)V}$ are given in (17) to (19); $\eta_{a\text{ to }c}^{(m)}$ are real to be determined by the requirement that the elastic fields satisfy the following conditions:

- $\vec{u}^{(0)(m)}(\vec{x})$ is continuous across the interface (actually we shall write this condition for the x_1 – component).
- $\oint_\Gamma du_2^{(0)(m)} = b$ for a closed contour in $x_1 x_2$ encircling the dislocation. We may take for Γ a square of side *a* centred at the origin and travelled in the direction of the corkscrew advancing in the positive x_3 – direction.
- The stresses $\sigma_{ij}^{(0)(m)}$ are continuous at the crossing of the interface, i.e. $\sigma_{ij}^{(0)(1)}(x_1, x_2 = 0, x_3) = \sigma_{ij}^{(0)(2)}(x_1, x_2 = 0, x_3)$.
- $\vec{u}^{(0)(m)W}(\vec{x})$ vanish far from the interface (i.e. when $|x_2| \to \infty$).

It can be seen that all the stresses involved in $(\sigma)^{(0)(m)\infty}$ and $(\sigma)_{a\ \text{to}\ c}^{(0)(m)V}$ vanish at infinity. Under such conditions above, $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ correspond to an interface straight edge dislocation. Next, we express the quantities involved in the conditions above and proceed to satisfy these.

$$u_1^{(0)(1)}(x_1, x_2 = 0, x_3) = u_1^{(0)(2)}(x_1, x_2 = 0, x_3) \Rightarrow$$

$$\frac{1-2\nu_m}{2\mu_m}\left\{C_m - \eta_a^{(m)}(-1)^{m-1} C_m iQ_a - \eta_b^{(m)}(-1)^{m-1} 2iQ_b \right.$$
$$\left. - \eta_c^{(m)}(-1)^{m-1}\frac{2i(Q_c - \nu_m \overline{V}_c)}{\nu_m(1-2\nu_m)}\right\} \equiv e_1;$$

$$\int_\Gamma du_2^{(0)(m)} = b \Rightarrow$$

$$\eta_a^{(m)} iQ_a + \eta_b^{(m)}\frac{2iQ_b}{C_m} + \eta_c^{(m)}\frac{2\pi i}{b\mu_m}\left(\frac{2(1-2\nu_m)Q_c}{\nu_m} + \overline{V}_c\right) \equiv e_2;$$

$$\sigma_{22}^{(0)(1)} = \sigma_{22}^{(0)(2)} \Rightarrow$$

$$C_m - \eta_a^{(m)}(-1)^{m-1} C_m iQ_a - \eta_b^{(m)}(-1)^{m-1} 2iQ_b$$
$$- \eta_c^{(m)}(-1)^{m-1} 2i\left(\frac{(1-2\nu_m)Q_c + \nu_m \overline{V}_c}{\nu_m}\right) \equiv e_3;$$

$$\sigma_{12}^{(0)(1)} = \sigma_{12}^{(0)(2)} \Rightarrow$$

$$\eta_c^{(m)} = \eta_c \equiv e_4;$$

$$\sigma_{33}^{(0)(1)} = \sigma_{33}^{(0)(2)} \Rightarrow$$

$$2\nu_m\left\{C_m - \eta_a^{(m)}(-1)^{m-1}C_m iQ_a - \eta_b^{(m)}(-1)^{m-1}2iQ_b - \eta_c^{(m)}(-1)^{m-1}2i\frac{Q_c}{\nu_m}\right\} \equiv e_5;$$

$$\sigma_{11}^{(0)(1)} = \sigma_{11}^{(0)(2)} \Rightarrow$$

$$C_m - \eta_a^{(m)}(-1)^{m-1}C_m iQ_a - \eta_b^{(m)}(-1)^{m-1}2iQ_b - \eta_c^{(m)}(-1)^{m-1}2i\left(\frac{(1+2\nu_m)Q_c - \nu_m\overline{V}_c}{\nu_m}\right) \equiv e_6;$$

$\vec{u}^{(0)(m)W}(\vec{x})$ vanishes when $|x_2| \to \infty \Rightarrow$

$$\eta_a^{(m)}\nu_m C_m iQ_a - \eta_b^{(m)}\nu_m 2iQ_b + \eta_c^{(m)}2iQ_c = 0 \equiv e_7; \quad (22)$$

where all e_i are constant with $m = 1$ and 2 ;for the various stress conditions above, we restrict ourselves to terms with the spatial function $(1/x_1)$ only.

III-1-3. Satisfying boundary conditions

We are concerned with finding the appropriate expressions for $\eta_{a \text{ to } c}^{(m)}$ that satisfy the boundary conditions (22). We first recognized that $\sigma_{12}^{(0)(m)}$ (20) takes on the interface the very simple form involving only $\eta_c^{(m)}$

$$\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3) = -\eta_c^{(m)}2\pi i(2Q_c - \overline{V}_c)\delta(x_1). \quad (23)$$

$\sigma_{12}^{(0)(1)} = \sigma_{12}^{(0)(2)}$ on the interface leads to $\eta_c^{(m)} = \eta_c \equiv e_4$ constant with *m* (22). A number of expressions for $\eta_c^{(m)}$ can be extracted from (22), but only one value leaves $\sigma_{12}^{(0)(m)}$ (23) unchanged on inverting the elastic constants. It is obtained as

$$\eta_c = \frac{2b\mu_1\mu_2[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]}{[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)]2\pi i(2Q_c - \overline{V}_c)}. \quad (24)$$

This can be arrived at from different routes involving different equations in (22). Only one route is presented as follows. $\eta_c^{(2)}$ can be isolated using (1) e_3 and e_6 in (22), (2) e_1 and e_3 and, (3) e_1 and e_2; (1)=(2) and (1)=(3) provide, respectively, the following equations :

$$C_1 - \eta_a^{(1)}C_1 iQ_a - \eta_b^{(1)}2iQ_b$$

$$+\eta_c \frac{2i\{\nu_1(\mu_2-\mu_1)\overline{V_c}+[-\mu_2+\mu_1(1+2\nu_1-2\nu_2)]Q_c\}}{\nu_1[\mu_2(1-2\nu_1)-\mu_1(1-2\nu_2)]}=0 \qquad (25)$$

$$2(1-\nu_1)(1-2\nu_2)C_1-b_1^*(C_1-\eta_a^{(1)}C_1iQ_a-\eta_b^{(1)}2iQ_b)$$
$$+\eta_c\frac{i}{\nu_1\mu_2}\left(2Q_cb_2^*-\nu_1\overline{V_c}[\mu_2-\mu_1(3-4\nu_2)]\right)=0 \qquad (26)$$

where

$$b_1^*=(1-\nu_1)(1-2\nu_2)+(1-\nu_2)(1-2\nu_1)\,,$$
$$b_2^*=\mu_2(2-2\nu_1-3\nu_2+4\nu_1\nu_2)-\mu_1\nu_1(3-4\nu_2)\,.$$

Both equations (25) and (26) yield η_c (24). Using (24), equations (25) and e_7 (22) allow $\eta_a^{(1)}$ and $\eta_b^{(1)}$ to be calculated; then to reach $\eta_a^{(2)}$ and $\eta_b^{(2)}$, we may associate to e_7 (22), the following equation (27) obtained in the same way as in (25) :

$$C_2+\eta_a^{(2)}C_2iQ_a+\eta_b^{(2)}2iQ_b$$
$$+\eta_c\frac{2i\{\nu_2(\mu_2-\mu_1)\overline{V_c}+[\mu_1-\mu_2(1-2\nu_1+2\nu_2)]Q_c\}}{\nu_2[\mu_1(1-2\nu_2)-\mu_2(1-2\nu_1)]}=0\,. \qquad (27)$$

We have

$$\eta_b^{(1)}-\eta_b^{(2)}=\frac{C_1+C_2}{C_1-C_2}=\frac{\mu_1(1-\nu_2)+\mu_2(1-\nu_1)}{\mu_1(1-\nu_2)-\mu_2(1-\nu_1)}$$
$$\eta_b^{(2)}=\frac{1}{C_2-C_1}\left(C_2+\frac{2b\mu_1\mu_2(\mu_1-\mu_2)}{\pi[\mu_1^2(3-4\nu_2)-\mu_2^2(3-4\nu_1)]}\right). \qquad (28)$$

$\eta_a^{(m)}$ depends on elastic constants, this is denoted as $\eta_a^{(m)}=\eta_a^{(m)}(\nu_1,\nu_2;\mu_1,\mu_2)$; by inverting the elastic constants, this becomes $\eta_a^{(m)}(\nu_2,\nu_1;\mu_2,\mu_1)$. We have the following results:

$$\eta_a^{(1)}(\nu_1,\nu_2;\mu_1,\mu_2)+\eta_a^{(2)}(\nu_2,\nu_1;\mu_2,\mu_1)=-\frac{i}{Q_a}\,;$$
$$\eta_a^{(2)}(\nu_1,\nu_2;\mu_1,\mu_2)=\frac{1}{2iQ_a}\left(-1+\frac{Nu}{De}\right) \qquad (29)$$

where

$$Nu = 2b\mu_1\left(\nu_1(1-\nu_2)d_1^*\mu_1^2 + \nu_2(1-\nu_1)e_2^*\mu_2^2 + \mu_1\mu_2\left[\nu_1(1-\nu_2)e_1^* + \nu_2(1-\nu_1)d_2^*\right]\right)$$

$$De = \pi(1-\nu_1)(\nu_1-\nu_2)(\nu_1 C_1 + \nu_2 C_2)\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]$$

in which

$$d_1^* = \nu_2 - 5\nu_1 + 8\nu_1\nu_2 ; \quad d_2^* = \nu_2 + 3\nu_1 - 8\nu_1\nu_2 ,$$
$$e_1^* = 5\nu_1 - \nu_2 - 8\nu_1^2 ; \quad e_2^* = -3\nu_1 - \nu_2 + 8\nu_1^2 .$$

In summary, $\eta_{a \text{ to } c}^{(m)}$ are determined by using (24), (28 and 29).

III-1-4. Perfect elastic fields

The elastic fields due to an interface straight edge dislocation ($\vec{b} = (0, b, 0)$) parallel to the x_3 – direction at the origin, are given in (20 and 21) with values of $\eta_{a \text{ to } c}^{(m)}$ calculated from (24), (28 and 29), respectively. We first display special values taken on the interface by quantities $\sigma_{12}^{(0)(m)}$ and $\sigma_{22}^{(0)(m)}$ that are frequently involved in the analyses of the propagation of the interface crack loaded in tension. $\sigma_{12}^{(0)(m)}$ is obtained from (23 and 24) as

$$\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3) = -\frac{2b\mu_1\mu_2\left[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)\right]}{\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]}\delta(x_1) , \tag{30}$$

$m = 1$ and 2. This quantity is unchanged by inverting the elastic constants. From (22), $\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3)$ is first written as

$$\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3) = \frac{e_3}{x_1} . \tag{31}$$

The calculation can be performed with *m*=1. Introducing in (31) the value of $(C_1 - \eta_a^{(1)} C_1 iQ_a - \eta_b^{(1)} 2iQ_b)$ taken from (25) and making use of the value of η_c (24), we obtain

$$\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3) = \frac{4b\mu_1\mu_2\left[\mu_1(1-\nu_2) - \mu_2(1-\nu_1)\right]}{\pi\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]}\frac{1}{x_1} \tag{32}$$

$\sigma_{22}^{(0)(m)}$ (32) is unchanged by inverting the elastic constants. Similarly, other elastic fields take, on the interface, the following values :

$$u_1^{(0)(m)}(x_1, x_2 = 0, x_3) = -\frac{2b\mu_1\mu_2(\nu_1 - \nu_2)}{\pi\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]}\ln|x_1|$$

$$\sigma_{11}^{(0)(m)}(x_1, x_2 = 0, x_3) = -\frac{4b\mu_1\mu_2(\nu_1\mu_2 - \nu_2\mu_1)}{\pi\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]}\frac{1}{x_1}$$

$$\sigma_{33}^{(0)(m)}(x_1, x_2 = 0, x_3) = \frac{4b\nu_m\mu_1\mu_2(\mu_1 - \mu_2)}{\pi\left[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)\right]}\frac{1}{x_1}\ . \tag{33}$$

III-2. Displacement and stress fields for an interface sinusoidal (glide-type) edge dislocation

III-2-1. Partial elastic oscillating fields

Four values for $(\overline{\alpha}_2^{(m)}(\kappa_n), \overline{\beta}_2^{(m)}(\kappa_n))$ are extracted from (15); these are:

(a) $$\overline{\alpha}_2^{(m)}(\kappa_n) = \frac{\xi_n a_{1a}^{(m)}}{2}\frac{k_1}{(k_1^2+\kappa_n^2)^{3/2}} \equiv \overline{\alpha}_{2a}^{A_n(m)},\quad \overline{\beta}_2^{(m)}(\kappa_n) = 0 \equiv \overline{\beta}_{2a}^{A_n(m)};$$

(b) $$\overline{\alpha}_2^{(m)}(\kappa_n) = (-1)^{m-1}\frac{iQ_b\xi_n k_1}{4\sqrt{k_1^2+\kappa_n^2}}\left(\frac{2}{k_1^2+\kappa_n^2} - \frac{1}{k_1^2+\rho_m\kappa_n^2} + \frac{1-4\nu_m}{k_1^2+\nu_m\kappa_n^2}\right) \equiv \overline{\alpha}_{2b}^{A_n(m)},$$

$$\overline{\beta}_2^{(m)}(\kappa_n) = -\frac{iQ_b\xi_n k_1}{2(k_1^2+\nu_m\kappa_n^2)} \equiv \overline{\beta}_{2b}^{A_n(m)};$$

(c) $$\overline{\alpha}_2^{(m)}(\kappa_n) = \frac{i\xi_n}{2}\left(\frac{a_{1c}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}} + \frac{a_{2c}^{(m)}k_1}{(k_1^2+\kappa_n^2)^{3/2}} + \frac{a_{3c}^{(m)}k_1}{(k_1^2+\theta\kappa_n^2)\sqrt{k_1^2+\kappa_n^2}}\right) \equiv \overline{\alpha}_{2c}^{A_n(m)},$$

$$\overline{\beta}_2^{(m)}(\kappa_n) = \frac{i\rho_m\xi_n}{2}\left(\frac{b_{1c}}{k_1} - \frac{b_{2c}k_1}{k_1^2+\kappa_n^2} + \frac{b_{3c}k_1}{k_1^2+\theta\kappa_n^2}\right) \equiv \overline{\beta}_{2c}^{A_n(m)};$$

(d) $$\overline{\alpha}_2^{(m)}(\kappa_n) = \frac{i\xi_n}{2}\left(\frac{a_{1d}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}} + \frac{a_{2d}^{(m)}k_1}{(k_1^2+\kappa_n^2)^{3/2}}\right) \equiv \overline{\alpha}_{2d}^{A_n(m)},$$

$$\overline{\beta}_2^{(m)}(\kappa_n) = \frac{i\xi_n\overline{b}^{(m)}}{2}\left(\frac{1}{k_1} + \frac{k_1}{k_1^2+\kappa_n^2}\right) \equiv \overline{\beta}_{2d}^{A_n(m)} \tag{34}$$

where

$a_{1a}^{(m)} = (-1)^{m-1}\mu_m bC_v / 8\pi$, $\rho_m = \nu_1\delta_{m2} + \nu_2\delta_{m1}$ (δ_{ij}: Kronecker delta), $\theta = (\nu_1+\nu_2)/2\nu_1\nu_2$;

$$a_{1c}^{(m)} = (Q_b - Q_c)\big([\nu_2 - \nu_1 + (-1)^{m-1}4\nu_1\nu_2] + (-1)^{m-1}2(\nu_1+\nu_2)\big)/\,2(\nu_1+\nu_2)\,,$$
$$a_{2c}^{(m)} = \big((Q_c - Q_b)[\nu_2 - \nu_1 + (-1)^{m-1}4\nu_1\nu_2] + (-1)^{m-1}4\nu_1\nu_2 Q_c(\theta-1)\big)/\,4\nu_1\nu_2(\theta-1)$$
,
$$a_{3c}^{(m)} = [\nu_2 - \nu_1 + (-1)^{m-1}4\nu_1\nu_2][Q_b - Q_c - 2Q_c\theta(\theta-1)]\,/\,2(\nu_1+\nu_2)(\theta-1)\,;$$

$$b_{1c} = (Q_b - Q_c)/(\nu_1+\nu_2)\,,\quad b_{2c} = (Q_b - Q_c)/2\nu_1\nu_2(\theta-1)\,,$$
$$b_{3c} = [Q_b - Q_c - 2Q_c\theta(\theta-1)]\,/\,2\nu_1\nu_2\theta(\theta-1)\,;$$

$$a_{1d}^{(m)} = \frac{(Q_c - Q_b)(-\delta_{m1}[\nu_1 - \nu_2(1+4\nu_1)] + \delta_{m2}[\nu_2 - \nu_1(1+4\nu_2)])}{2[\nu_1(1-\nu_2) + \nu_2(1-\nu_1)]}\,,$$
$$a_{2d}^{(m)} = \frac{1}{2[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)]}\big(\delta_{m1}[Q_c(\nu_1+3\nu_2) + Q_b(\nu_1 - \nu_2(1+4\nu_1))]$$
$$- \delta_{m2}[Q_c(\nu_2+3\nu_1) + Q_b(\nu_2 - \nu_1(1+4\nu_2))]\big)\,;$$

$$\bar{b}^{(m)} = \rho_m(Q_c - Q_b)/[\nu_1(1-\nu_2) + \nu_2(1-\nu_1)]\,. \qquad (35)$$

Similarly to (16), none of these couples satisfies the entire conditions (15). For each couple, we give below the associated oscillating elastic fields $\vec{u}^{\,A_n(m)V}$ and $(\sigma)^{A_n(m)V}$ defined in Section 2.2. A superposition of these partial fields will provide the complete form of solution (to first order in ξ_n). The couple $(\bar{\alpha}_{2a}^{A_n(m)}, \bar{\beta}_{2a}^{A_n(m)})$ is obtained from (15 a to d) associated with the displacement. We have at position $\vec{x} = (x_1, x_2, x_3)$ $(\vec{u}^{\,A_n(m)V} \equiv \vec{u}_a^{\,A_n(m)V}, (\sigma)^{A_n(m)V} \equiv (\sigma)_a^{A_n(m)V})$:

$$u_{ia}^{A_n(m)V} = \frac{a_{1a}^{(m)}}{2\mu_m}\left\|\delta_{i1} + \delta_{i2} + \delta_{i3}\frac{\partial}{\partial x_3}\right\| A_n$$
$$\times\left(\frac{2\kappa_n K_1[\kappa_n r]}{r}\big(\delta_{i1}(-1)^m|x_2| + \delta_{i2}x_1\big) + (-1)^{m-1}\left\|\kappa_n^2\delta_{i1} + \delta_{i3}\frac{\partial}{\partial x_1}\right\|\Pi_1\right)$$
,
$$\sigma_{iia}^{A_n(m)V} = (-1)^m a_{1a}^{(m)} A_n\left((\delta_{i1} - \delta_{i2})\frac{\partial I_0}{\partial x_1} - \kappa_n^2(\delta_{i1} - \delta_{i3})\frac{\partial \Pi_1}{\partial x_1}\right),$$
$$\sigma_{12a}^{A_n(m)V} = -a_{1a}^{(m)} A_n\left(\mathrm{sgn}(x_2)\frac{\partial I_0}{\partial x_2} + 2\kappa_n^2 K_0[\kappa_n r]\right),$$

$$\sigma_{j3a}^{A_n(m)V} = a_{1a}^{(m)} \frac{\partial A_n}{\partial x_3}\left(\delta_{j1}(-1)^m (I_0 - \kappa_n^2 \Pi_1) + \delta_{j2} \frac{2\kappa_n x_1 K_1}{r}\right); \qquad (36)$$

$$\Pi_z = \int_{-\infty}^{\infty} \frac{e^{(-1)^m \sqrt{k_1^2+\kappa_n^2}\, x_2}}{k_1^2 + z\kappa_n^2} e^{ik_1x_1} dk_1 \equiv \int_{-\infty}^{\infty} \frac{e^{-\sqrt{k_1^2+\kappa_n^2}\,|x_2|}}{k_1^2 + z\kappa_n^2} e^{ik_1x_1} dk_1,$$

$$I_0 = \int_{-\infty}^{\infty} e^{(-1)^m \sqrt{k_1^2+\kappa_n^2}\, x_2} e^{ik_1x_1} dk_1 \equiv \int_{-\infty}^{\infty} e^{-\sqrt{k_1^2+\kappa_n^2}\,|x_2|} e^{ik_1x_1} dk_1 = \frac{2\kappa_n |x_2|}{r} K_1.$$

Terms in brackets $\| \; \|$ are operators acting on A_n and Π_1 (i.e. Π_z with z = 1), separately; $r^2 = x_1^2 + x_2^2$ and subscripts i= 1 to 3 and j= 1 and 2; $K_n[x]$ is the nth-order modified Bessel function usually so denoted and δ_{ij} is the Kronecker delta. We stress that the various integrations (such as in I_0 and Π_z) performed in the present study are given for spatial positions satisfying the condition $(-1)^m x_2 < 0$ (*i.e.* $(-1)^{m-1} = \mathrm{sgn}(x_2)$) with $m = 1$ when $x_2 > \xi_n \sin \kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin \kappa_n x_3$ (half-space 2). However, this makes no difference in the elastic fields to first order in ξ_n. The pair $(\overline{\alpha}_{2b}^{A_n(m)}, \overline{\beta}_{2b}^{A_n(m)})$ is obtained using (15 *e* to *h*) associated with the stresses. We have (using the similar notations)

$$u_{1b}^{A_n(m)V} = -\frac{iQ_b A_n}{4\mu_m}\left(2\left\|2(1-\nu_m) + x_2 \frac{\partial}{\partial x_2}\right\| I_0 - 2\kappa_n^2 \Pi_1 + \rho_m \kappa_n^2 \Pi_{\rho_m} - \nu_m \kappa_n^2 \left\|3 - 4\nu_m + 2x_2 \frac{\partial}{\partial x_2}\right\| \Pi_{\nu_m}\right),$$

$$u_{2b}^{A_n(m)V} = \frac{iQ_b A_n}{4\mu_m}\left(\frac{2}{x_2}\left\|x_1 + x_2^2 \frac{\partial}{\partial x_1}\right\| I_0 - \frac{\partial^2 \Pi_{\rho_m}}{\partial x_1 \partial x_2} + \left\|2(1-\nu_m)\kappa_n^2 x_2 - (3-4\nu_m)\frac{\partial}{\partial x_2}\right\| \frac{\partial \Pi_{\nu_m}}{\partial x_1}\right),$$

$$u_{3b}^{A_n(m)V} = \frac{iQ_b}{4\mu_m} \frac{\partial A_n}{\partial x_3} \frac{\partial}{\partial x_1}\left(2\Pi_1 - \Pi_{\rho_m} + \left\|3 - 4\nu_m + 2x_2 \frac{\partial}{\partial x_2}\right\| \Pi_{\nu_m}\right),$$

$$\sigma_{iib}^{A_n(m)V} = -\frac{iQ_b A_n}{2} \frac{\partial}{\partial x_1}\left(2\left\|2(\delta_{i1} + \nu_m \delta_{i3}) + (\delta_{i1} - \delta_{i2}) x_2 \frac{\partial}{\partial x_2}\right\| I_0\right.$$

$$+ 2\kappa_n^2(-\delta_{i1} + \delta_{i3})\Pi_1 + \kappa_n^2[\rho_m\delta_{i1} + (1-\rho_m)\delta_{i2} - \delta_{i3}]\Pi_{\rho_m}$$

$$+ \kappa_n^2\left\|\nu_m\delta_{i1} + (1-\nu_m)\delta_{i2} + (3-4\nu_m^2)\delta_{i3} - 2[\nu_m\delta_{i1} + (1-\nu_m)\delta_{i2} - \delta_{i3}]x_2\frac{\partial}{\partial x_2}\right\|\Pi_{\nu_m}\Bigg)$$

$$\sigma_{12b}^{A_n(m)V} = -\frac{iQ_bA_n}{2}\left(4\,\mathrm{sgn}(x_2)\frac{\partial^2K_0}{\partial x_1^2} + 2x_2\left\|(1-\nu_m)\kappa_n^2 - \frac{\partial^2}{\partial x_1^2}\right\|I_0 + \rho_m\kappa_n^2\frac{\partial\Pi_{\rho_m}}{\partial x_2}\right.$$

$$\left. - \nu_m\kappa_n^2\left\|2(1-\nu_m)\kappa_n^2x_2 + \frac{\partial}{\partial x_2}\right\|\Pi_{\nu_m}\right),$$

$$\sigma_{13b}^{A_n(m)V} = -\frac{iQ_b}{2}\frac{\partial A_n}{\partial x_3}\left(2\left\|2(1-\nu_m) + x_2\frac{\partial}{\partial x_2}\right\|I_0 - 2\kappa_n^2\Pi_1\right.$$

$$\left. + \rho_m\kappa_n^2\Pi_{\rho_m} - \nu_m\kappa_n^2\left\|3 - 4\nu_m + 2x_2\frac{\partial}{\partial x_2}\right\|\Pi_{\nu_m}\right),$$

$$\sigma_{23b}^{A_n(m)V} = \frac{iQ_b}{2}\frac{\partial A_n}{\partial x_3}\frac{\partial}{\partial x_1}\left(-4\,\mathrm{sgn}(x_2)K_0 + 2x_2I_0 - \frac{\partial\Pi_{\rho_m}}{\partial x_2}\right.$$

$$\left. + \left\|2(1-\nu_m)\kappa_n^2x_2 + \frac{\partial}{\partial x_2}\right\|\Pi_{\nu_m}\right). \qquad (37)$$

The couple $(\overline{\alpha}_{2c}^{A_n(m)}, \overline{\beta}_{2c}^{A_n(m)})$ is calculated from (15 *i* to *l*) associated with stresses. We obtain, with the similar notations,

$$u_{1c}^{A_n(m)V} = \frac{iA_n}{2\mu_m}\left(-2b_{2c}\rho_m\kappa_n^2|x_2|K_0 + \sum_{s=1}^{3}\left[(-1)^m a_{sc}^{(m)} + (-1)^{s-1}b_{sc}\rho_m\left\|1 + x_2\frac{\partial}{\partial x_2}\right\|\right]I_0\right.$$

$$\left. + \kappa_n^2\left(\rho_m b_{2c} + (-1)^{m-1}a_{2c}^{(m)}\right)\Pi_1 + \theta\kappa_n^2\left[(-1)^{m-1}a_{3c}^{(m)} - \rho_m b_{3c}\left\|1 + x_2\frac{\partial}{\partial x_2}\right\|\right]\Pi_\theta\right),$$

$$u_{2c}^{A_n(m)V} = \frac{iA_n}{2\mu_m}\left(\left[\left((-1)^{m-1}a_{2c}^{(m)} - 2\rho_m(1-2\nu_m)b_{2c}\right)\frac{x_1}{x_2} + \rho_m\sum_{s=1}^{3}\left\|(-1)^s b_{sc}x_2\frac{\partial}{\partial x_1}\right\|\right]I_0\right.$$

$$+ 2\kappa_n\,\mathrm{sgn}(x_1)\left[-b_{1c}\rho_m\kappa_n^2\,\mathrm{sgn}(x_2)x_2^2 + \left(a_{1c}^{(m)} + (-1)^{m-1}2\rho_m(1-2\nu_m)b_{1c}\right)\left\|1 + x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{J}$$

$$\left. + \left[\rho_m(\theta-1)b_{3c}\kappa_n^2x_2 + \left\|\left((-1)^{m-1}a_{3c}^{(m)} + 2\rho_m(1-2\nu_m)b_{3c}\right)\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial\Pi_\theta}{\partial x_1}\right),$$

$$u_{3c}^{A_n(m)V} = \frac{i}{2\mu_m}\frac{\partial A_n}{\partial x_3}\Bigg(b_{2c}\rho_m x_1 I_0 - 2\kappa_n \operatorname{sgn}(x_1)|x_2|\left[(-1)^m a_{1c}^{(m)} + \rho_m b_{1c}\left\|2 + x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}$$

$$+\left((-1)^{m-1}a_{2c}^{(m)} + \rho_m b_{2c}\right)\frac{\partial \Pi_1}{\partial x_1} - \left[(-1)^m a_{3c}^{(m)} + \rho_m b_{3c}\left\|1 + x_2\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial \Pi_\theta}{\partial x_1}\Bigg),$$

$$\sigma_{11c}^{A_n(m)V} = -iA_n\Bigg(-\rho_m b_{2c}\kappa_n^2 x_1 I_0 + \Bigg[\sum_{s=1}^{3}\left((-1)^{m-1}a_{sc}^{(m)} + \rho_m(1+2\nu_m)(-1)^s b_{sc}\right)$$

$$+\rho_m|x_2|\left\|b_{3c} - (b_{1c} - b_{2c})\operatorname{sgn}(x_2)\frac{\partial}{\partial x_2}\right\|\Bigg]\frac{\partial I_0}{\partial x_1} - 4\nu_m\rho_m b_{1c}\kappa_n^3|x_2|\operatorname{sgn}(x_1)\overline{\overline{J}}$$

$$+\kappa_n^2\left((-1)^m a_{2c}^{(m)} - \rho_m b_{2c}\right)\frac{\partial \Pi_1}{\partial x_1}$$

$$-\kappa_n^2\left[(-1)^{m-1}a_{3c}^{(m)}\theta + \rho_m b_{3c}\left\|2\nu_m - \theta\left(1 + 2\nu_m + x_2\frac{\partial}{\partial x_2}\right)\right\|\right]\frac{\partial \Pi_\theta}{\partial x_1}\Bigg),$$

$$\sigma_{22c}^{A_n(m)V} = iA_n\Bigg(\sum_{s=1}^{3}\left[(-1)^{m-1}a_{sc}^{(m)} + (-1)^{s-1}b_{sc}\rho_m\left\|1 - 2\nu_m - x_2\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial I_0}{\partial x_1}$$

$$+2\kappa_n^3\operatorname{sgn}(x_1)|x_2|\left[(-1)^{m-1}a_{1c}^{(m)} - \rho_m b_{1c}\left\|2\nu_m + x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}$$

$$+\kappa_n^2(1-\theta)\left[(-1)^{m-1}a_{3c}^{(m)} + \rho_m b_{3c}\left\|1 - 2\nu_m - x_2\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial \Pi_\theta}{\partial x_1}\Bigg),$$

$$\sigma_{33c}^{A_n(m)V} = -iA_n\Bigg(\rho_m\left\|b_{2c}\kappa_n^2 x_1 + 2\nu_m\sum_{s=1}^{3}(-1)^s b_{sc}\frac{\partial}{\partial x_1}\right\|I_0 - \kappa_n^2\left((-1)^m a_{2c}^{(m)} - b_{2c}\rho_m\right)\frac{\partial \Pi_1}{\partial x_1}$$

$$-2\kappa_n^3\operatorname{sgn}(x_1)|x_2|\left[(-1)^m a_{1c}^{(m)} + b_{1c}\rho_m\left\|2(1+\nu_m) + x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}$$

$$-\kappa_n^2\left[(-1)^m a_{3c}^{(m)} + \rho_m b_{3c}\left\|1 + 2\nu_m(1-\theta) + x_2\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial \Pi_\theta}{\partial x_1}\Bigg)$$

,

$$\sigma_{12c}^{A_n(m)V} = iA_n\Bigg(2\kappa_n^2\left(-a_{2c}^{(m)} + (-1)^m 2\nu_m\rho_m b_{2c}\right)K_0 + \Bigg[\rho_m\kappa_n^2\left(b_{1c} + b_{3c}(1-\theta)\right)x_2$$

$$+\sum_{s=1}^{3}\left\|\left((-1)^m a_{sc}^{(m)} + 2\rho_m\nu_m(-1)^{s-1}b_{sc}\right)\frac{\partial}{\partial x_2} + (-1)^s b_{sc}\rho_m x_2\frac{\partial^2}{\partial x_1^2}\right\|\Bigg]I_0$$

$$+\theta\kappa_n^2\left[\rho_m(\theta-1)b_{3c}\kappa_n^2x_2+\left\|\left((-1)^{m-1}a_{3c}^{(m)}-2\rho_m\nu_m b_{3c}\right)\frac{\partial}{\partial x_2}\right\|\right]\Pi_\theta\Bigg),$$

$$\sigma_{13c}^{A_n(m)V}=i\frac{\partial A_n}{\partial x_3}\Bigg(-2b_{2c}\rho_m\kappa_n^2|x_2|K_0+\sum_{s=1}^{3}\left[(-1)^m a_{sc}^{(m)}+\rho_m(-1)^{s-1}b_{sc}\left\|1+x_2\frac{\partial}{\partial x_2}\right\|\right]I_0$$

$$+\kappa_n^2\left(\rho_m b_{2c}+(-1)^{m-1}a_{2c}^{(m)}\right)\Pi_1+\theta\kappa_n^2\left[(-1)^{m-1}a_{3c}^{(m)}-\rho_m b_{3c}\left\|1+x_2\frac{\partial}{\partial x_2}\right\|\right]\Pi_\theta\Bigg),$$

$$\sigma_{23c}^{A_n(m)V}=i\frac{\partial A_n}{\partial x_3}\Bigg(\left[\left((-1)^{m-1}a_{2c}^{(m)}+2\rho_m\nu_m b_{2c}\right)\frac{x_1}{x_2}+\rho_m\sum_{s=1}^{3}\left\|(-1)^s b_{sc}x_2\frac{\partial}{\partial x_1}\right\|\right]I_0$$

$$+2\kappa_n\,\mathrm{sgn}(x_1)\left[-b_{1c}\rho_m\kappa_n^2\,\mathrm{sgn}(x_2)x_2^2+\left(a_{1c}^{(m)}+(-1)^m 2\rho_m\nu_m b_{1c}\right)\left\|1+x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}$$

$$+\left[\rho_m(\theta-1)b_{3c}\kappa_n^2x_2+\left\|\left((-1)^{m-1}a_{3c}^{(m)}-2\rho_m\nu_m b_{3c}\right)\frac{\partial}{\partial x_2}\right\|\right]\frac{\partial\Pi_\theta}{\partial x_1}\Bigg),$$

$$\overline{\overline{J}}=\int_{|x_1|}^{\infty}\frac{K_1[\kappa_n\sqrt{x^2+x_2^2}\,]}{\sqrt{x^2+x_2^2}}dx\,. \qquad (38)$$

The pair $(\overline{\alpha}_{2d}^{A_n(m)},\overline{\beta}_{2d}^{A_n(m)})$ is calculated from (15 m to p) associated with stresses. In a similar way, we obtain

$$u_{1d}^{A_n(m)V}=\frac{iA_n}{2\mu_m}\Bigg(2\overline{b}^{(m)}\kappa_n^2|x_2|K_0+\left[(-1)^m\left(a_{1d}^{(m)}+a_{2d}^{(m)}\right)+2\overline{b}^{(m)}\left\|1+x_2\frac{\partial}{\partial x_2}\right\|\right]I_0$$

$$-\kappa_n^2\left((-1)^m a_{2d}^{(m)}+\overline{b}^{(m)}\right)\Pi_1\Bigg),$$

$$u_{2d}^{A_n(m)V}=\frac{iA_n}{2\mu_m}\Bigg(\left\|\left((-1)^{m-1}a_{2d}^{(m)}+2(1-2\nu_m)\overline{b}^{(m)}\right)\frac{x_1}{x_2}-2\overline{b}^{(m)}x_2\frac{\partial}{\partial x_1}\right\|I_0$$

$$-2\kappa_n\,\mathrm{sgn}(x_1)\left[\overline{b}^{(m)}\kappa_n^2\,\mathrm{sgn}(x_2)x_2^2+\left((-1)^m 2(1-2\nu_m)\overline{b}^{(m)}-a_{1d}^{(m)}\right)\left\|1+x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}\Bigg),$$

$$u_{3d}^{A_n(m)V}=\frac{i}{2\mu_m}\frac{\partial A_n}{\partial x_3}\Bigg(-\overline{b}^{(m)}x_1I_0+\left((-1)^{m-1}a_{2d}^{(m)}-\overline{b}^{(m)}\right)\frac{\partial\Pi_1}{\partial x_1}$$

$$-2\kappa_n\,\mathrm{sgn}(x_1)|x_2|\left[(-1)^m a_{1d}^{(m)}+\overline{b}^{(m)}\left\|2+x_2\frac{\partial}{\partial x_2}\right\|\right]\overline{\overline{J}}\Bigg),$$

$$\sigma_{11d}^{A_n(m)V} = iA_n\left(\kappa_n^2\left((-1)^{m-1}a_{2d}^{(m)} - \overline{b}^{(m)}\right)\frac{\partial \Pi_1}{\partial x_1} + 4\overline{b}^{(m)}\nu_m\kappa_n^3\,\mathrm{sgn}(x_1)\left|x_2\right|\overline{\overline{J}}\right.$$

$$\left. + \left[\left((-1)^m(a_{1d}^{(m)} + a_{2d}^{(m)}) + 2(1+2\nu_m)\overline{b}^{(m)}\right)\frac{\partial}{\partial x_1} - \overline{b}^{(m)}\left[x_1\kappa_n^2 - 2x_2\frac{\partial^2}{\partial x_1\partial x_2}\right]\right]I_0\right),$$

$$\sigma_{22d}^{A_n(m)V} = iA_n\left(\left[(-1)^{m-1}(a_{1d}^{(m)} + a_{2d}^{(m)}) + 2\overline{b}^{(m)}\left[1 - 2\nu_m - x_2\frac{\partial}{\partial x_2}\right]\right]\frac{\partial I_0}{\partial x_1}\right.$$

$$\left. + 2\kappa_n^3\,\mathrm{sgn}(x_1)\left|x_2\right|\left[(-1)^{m-1}a_{1d}^{(m)} - \overline{b}^{(m)}\left[2\nu_m + x_2\frac{\partial}{\partial x_2}\right]\right]\overline{\overline{J}}\right),$$

$$\sigma_{33d}^{A_n(m)V} = iA_n\left(\overline{b}^{(m)}\left[\kappa_n^2 x_1 + 4\nu_m\frac{\partial}{\partial x_1}\right]I_0 + \kappa_n^2\left((-1)^m a_{2d}^{(m)} + \overline{b}^{(m)}\right)\frac{\partial \Pi_1}{\partial x_1}\right.$$

$$\left. + 2\kappa_n^3\,\mathrm{sgn}(x_1)\left|x_2\right|\left[(-1)^m a_{1d}^{(m)} + \overline{b}^{(m)}\left[2(1+\nu_m) + x_2\frac{\partial}{\partial x_2}\right]\right]\overline{\overline{J}}\right),$$

$$\sigma_{12d}^{A_n(m)V} = iA_n\left(2\kappa_n^2\left((-1)^{m-1}2\nu_m\overline{b}^{(m)} - a_{2d}^{(m)}\right)K_0 + \left[\left((-1)^m(a_{1d}^{(m)} + a_{2d}^{(m)}) + 4\nu_m\overline{b}^{(m)}\right)\frac{\partial}{\partial x_2}\right.\right.$$

$$\left.\left. + \overline{b}^{(m)}x_2\left[\kappa_n^2 - 2\frac{\partial^2}{\partial x_1^2}\right]\right]I_0\right),$$

$$\sigma_{13d}^{A_n(m)V} = i\frac{\partial A_n}{\partial x_3}\left(\left[(-1)^m(a_{1d}^{(m)} + a_{2d}^{(m)}) + 2\overline{b}^{(m)}\left[1 + x_2\frac{\partial}{\partial x_2}\right]\right]I_0\right.$$

$$\left. - \kappa_n^2\left((-1)^m a_{2d}^{(m)} + \overline{b}^{(m)}\right)\Pi_1 + 2\overline{b}^{(m)}\kappa_n^2\left|x_2\right|K_0\right),$$

$$\sigma_{23d}^{A_n(m)V} = i\frac{\partial A_n}{\partial x_3}\left(\left[(-1)^{m-1}a_{2d}^{(m)}\frac{x_1}{x_2} - 2\overline{b}^{(m)}\left[\nu_m\frac{x_1}{x_2} + x_2\frac{\partial}{\partial x_1}\right]\right]I_0\right.$$

$$\left. + 2\kappa_n\,\mathrm{sgn}(x_1)\left[-\overline{b}^{(m)}\kappa_n^2\,\mathrm{sgn}(x_2)x_2^2 + \left((-1)^m 2\nu_m\overline{b}^{(m)} + a_{1d}^{(m)}\right)\left[1 + x_2\frac{\partial}{\partial x_2}\right]\right]\overline{\overline{J}}\right). \quad (39)$$

III-2-2. Boundary conditions and corresponding elastic fields

To first order with respect to the perturbation A_n (i.e. ξ_n), the elastic fields $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ of an interface sinusoidal edge dislocation, ***Figure 1***, as deduced from (3), (6), (13), and (20), may be written as

$$\vec{u}^{(m)} = \vec{u}^{(0)(m)} + \vec{u}^{A_n(m)} \; ; \quad (\sigma)^{(m)} = (\sigma)^{(0)(m)} + (\sigma)^{A_n(m)} \qquad (40)$$

$\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$ (Section 3.1) correspond to the fields of a straight edge dislocation lying on a planar interface; $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ are oscillating expressions proportional to either the sinusoid $A_n(x_3)$ or its spatial derivative $\partial A_n / \partial x_3$ in the forms

$$\vec{u}^{A_n(m)} = \vec{u}^{A_n(m)\infty} - \vec{u}^{A_n(m)W} \; ; \quad (\sigma)^{A_n(m)} = (\sigma)^{A_n(m)\infty} - (\sigma)^{A_n(m)W} \qquad (41)$$

with

$$\vec{u}^{A_n(m)W} = \eta_a^{A_n(m)} \vec{u}_a^{A_n(m)V} + \eta_b^{A_n(m)} \vec{u}_b^{A_n(m)V} + \eta_c^{A_n(m)} \vec{u}_c^{A_n(m)V} + \eta_d^{A_n(m)} \vec{u}_d^{A_n(m)V}$$

$$(\sigma)^{A_n(m)W} = \eta_a^{A_n(m)} (\sigma)_a^{A_n(m)V} + \eta_b^{A_n(m)} (\sigma)_b^{A_n(m)V} + \eta_c^{A_n(m)} (\sigma)_c^{A_n(m)V} + \eta_d^{A_n(m)} (\sigma)_d^{A_n(m)V} \qquad (42)$$

$\vec{u}_{a \text{ to } d}^{A_n(m)V}$ and $(\sigma)_{a \text{ to } d}^{A_n(m)V}$ are given in (36) to (39) (for $\vec{u}_{a \text{ to } d}^{A_n(m)\infty}$ and $(\sigma)_{a \text{ to } d}^{A_n(m)\infty}$, see (6)) ; $\eta_{a \text{ to } d}^{A_n(m)}$ are real to be determined by the requirement that the elastic fields satisfy the following conditions:

- $\vec{u}^{(m)}$ (hence $\vec{u}^{A_n(m)}$) is continuous across the interface. Actually we shall write this condition for the x_1 and x_3 components (*i.e.* $u_1^{A_n(m)}$ and $u_3^{A_n(m)}$).
- $\oint_\Gamma d\vec{u}^{(m)} = \vec{b}$ for a closed contour Γ encircling the dislocation . With the continuity condition above, imposed on $u_1^{(m)}$ and $u_3^{(m)}$, it is sufficient to satisfy this condition for the x_2 – component $u_2^{(m)}$ only; Because $\oint_\Gamma du_2^{(0)(m)} = b$ (see Section 3.1.2), we shall just have to satisfy the condition $\oint_\Gamma du_2^{A_n(m)} = 0$. Again, we may take for Γ a square of side *a* in $x_1 x_2$ – planes surrounding the dislocation and travelled in the direction of the corkscrew advancing in the positive x_3 – direction.
- $\sigma_{ij}^{(m)}$ (hence $\sigma_{ij}^{A_n(m)}$) are continuous at the crossing of the interface, *i.e.*

$\sigma_{ij}^{A_n(1)}(P_s) = \sigma_{ij}^{A_n(2)}(P_s)$.

- $\vec{u}^{(m)W} = \vec{u}^{(0)(m)W} + \vec{u}^{A_n(m)W}$ (hence $\vec{u}^{A_n(m)W}$) vanish far from the interface (i.e. when $|x_2| \to \infty$). This condition is automatically fulfilled because $\vec{u}^{(0)(m)W}$ tends to zero when $|x_2| \to \infty$ (see Section 3.1.2) and $u_{ij}^{A_n(m)V}$ (*i*= 1 to 3 and *j*= *a* to *d*) equally (see their various expressions in (36) to (39)).

It can be seen that all the stresses involved in $(\sigma)^{A_n(m)\infty}$ and $(\sigma)_{a\ \text{to}\ d}^{A_n(m)V}$ are vanishing at infinity. Under such conditions above, $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ correspond to an interface sinusoidal edge dislocation (up to terms of first order (zero and one) with respect to the perturbation A_n). Next, we express the quantities involved in the various conditions above and then proceed to satisfy these. Before displaying the equations describing the above boundary conditions, we stress the followings: (1) For point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ on the interface, expanding the elastic fields to terms of first order with respect to $x_2 = \xi$, *i.e.*

$$\vec{u}^{A_n(m)}(x_1, x_2 = \xi, x_3) = \vec{u}^{A_n(m)}(x_1, 0, x_3) + \frac{\partial \vec{u}^{A_n(m)}}{\partial x_2}(x_1, 0, x_3)\xi$$

$$(\sigma)^{A_n(m)}(x_1, x_2 = \xi, x_3) = (\sigma)^{A_n(m)}(x_1, 0, x_3) + \frac{\partial (\sigma)^{A_n(m)}}{\partial x_2}(x_1, 0, x_3)\xi ,$$

reveals that second terms (on the right of the equations above) are proportional to ξ_n^2 because the elastic fields are proportional to $A_n = \xi_n \sin \kappa_n x_3$ or $\partial A_n / \partial x_3$ Hence continuity conditions for the elastic fields on the interface will be imposed on the first terms only, *i.e.*

$$\vec{u}^{A_n(1)}(x_1, x_2 = 0, x_3) = \vec{u}^{A_n(2)}(x_1, 0, x_3)$$

$$(\sigma)^{A_n(1)}(x_1, 0, x_3) = (\sigma)^{A_n(2)}(x_1, 0, x_3) .$$

(2) In most cases, the elastic fields on the interface are exponentially decreasing functions with x_1; these are bounded functions. Under such conditions, we have considered their linear forms with respect to x_1 and posed the coefficients proportional to x_1 constant with *m*. However, for $\sigma_{12}^{A_n(m)}$ and $\sigma_{23}^{A_n(m)}$, in addition to bounded functions, there exist terms proportional to $K_0[\kappa_n |x_1|]$ and $K_1[\kappa_n |x_1|]$ (modified Bessel functions, unbounded); both (more precisely $K_1[\kappa_n |x_1|]/|x_1|$ and $K_0[\kappa_n |x_1|]$) appear in $\sigma_{12}^{A_n(m)}$ and $K_1[\kappa_n |x_1|]$ only in $\sigma_{23}^{A_n(m)}$.

For both stresses, we have imposed the coefficients of the unbounded functions constant with *m* separately; the coefficient of $K_0[\kappa_n|x_1|]$ in $\sigma_{12}^{A_n(m)}$ correspond to that of $K_1[\kappa_n|x_1|]$ in $\sigma_{23}^{A_n(m)}$. The contributions of their associated bounded functions are not taken into account. It is interesting to mention that there is no singularity of the type $1/x_1$ in $\sigma_{12}^{A_n(m)}$. Hence, this stress does not contribute to the crack extension force when a crack (*i.e.* a continuous distribution of infinitesimal dislocations) in the interface is loaded in tension. On the contrary, because the singularity in $K_1[\kappa_n|x_1|]$ is of the $1/x_1$ type, $\sigma_{23}^{A_n(m)}$ does. (3) Corresponding to the condition $\oint_\Gamma du_2^{A_n(m)} = 0$, is written below the relation that cancels the coefficient of $K_1[\kappa_n a/2]$ only, in the mathematical expression of $\oint_\Gamma du_2^{A_n(m)}$; all other terms are proportional to the side *a* of the square Γ that may be neglected for small values of *a*. We may write :

$$u_1^{A_n(1)}(x_1, x_2 = 0, x_3) = u_1^{A_n(2)}(x_1, x_2 = 0, x_3) \Rightarrow$$

$$\frac{1}{\mu_m}\Big\{\eta_a^{A_n(m)}(-1)^{m-1}2a_{1a}^{(m)} + \eta_b^{A_n(m)} iQ_b[2 + \nu_m(3 - 4\nu_m) - \rho_m]$$
$$+ \eta_c^{A_n(m)} 2i\left[\rho_m b_{2c} + (-1)^{m-1}a_{2c}^{(m)} + \theta\left((-1)^{m-1}a_{3c}^{(m)} - \rho_m b_{3c}\right)\right]$$
$$+ \eta_d^{A_n(m)} 2i\left[(-1)^{m-1}a_{2d}^{(m)} - \overline{b}^{(m)}\right]\Big\} \equiv e_1^{A_n};$$

$$u_3^{A_n(1)} = u_3^{A_n(2)} \Rightarrow$$

$$\frac{1}{\mu_m}\Big\{\eta_a^{A_n(m)}(-1)^{m-1}2ia_{1a}^{(m)} + \eta_b^{A_n(m)}Q_b\left[-2 + \sqrt{\rho_m} - \sqrt{\nu_m}(3 - 4\nu_m)\right]$$
$$+ \eta_c^{A_n(m)} 2\left[-\rho_m b_{2c} + (-1)^m a_{2c}^{(m)} + \sqrt{\theta}\left((-1)^m a_{3c}^{(m)} + \rho_m b_{3c}\right)\right]$$
$$+ \eta_d^{A_n(m)} 2\left[(-1)^m a_{2d}^{(m)} + \overline{b}^{(m)}\right]\Big\} \equiv e_2^{A_n};$$

$$\oint_\Gamma du_2^{A_n(m)} = 0 \Rightarrow$$

$$(-1)^m bC_\nu + \eta_a^{A_n(m)}(-1)^{m-1}bC_\nu + \eta_b^{A_n(m)}(-1)^{m-1}\frac{8\pi iQ_b}{\mu_m}$$
$$+ \eta_c^{A_n(m)}\frac{8\pi i}{\mu_m}\left[(-1)^m 2\rho_m(1 - 2\nu_m)b_{2c} + a_{2c}^{(m)}\right]$$
$$+ \eta_d^{A_n(m)}\frac{8\pi i}{\mu_m}\left[(-1)^{m-1}2(1 - 2\nu_m)\overline{b}^{(m)} + a_{2d}^{(m)}\right] \equiv e_3^{A_n};$$

$$\sigma_{11}^{A_n(1)} = \sigma_{11}^{A_n(2)} \Rightarrow$$
$$\eta_a^{A_n(m)}(-1)^{m-1}2a_{1a}^{(m)} + \eta_b^{A_n(m)} iQ_b\left[2-\rho_m^{3/2}-\nu_m^{3/2}\right]$$
$$+\eta_c^{A_n(m)}2i\left[\rho_m b_{2c} + (-1)^{m-1}a_{2c}^{(m)} + \sqrt{\theta}\left((-1)^{m-1}a_{3c}^{(m)}\theta + \rho_m b_{3c}\left(2\nu_m(1-\theta)-\theta\right)\right)\right]$$
$$+\eta_d^{A_n(m)}2i\left[(-1)^{m-1}a_{2d}^{(m)} - \overline{b}^{(m)}\right] \equiv e_4^{A_n};$$

$$\sigma_{22}^{A_n(1)} = \sigma_{22}^{A_n(2)} \Rightarrow$$
$$\eta_b^{A_n(m)} Q_b\left[(1-\nu_m)\sqrt{\nu_m} + (1-\rho_m)\sqrt{\rho_m}\right]$$
$$+\eta_c^{A_n(m)}2(1-\theta)\sqrt{\theta}\left[(-1)^m a_{3c}^{(m)} + (-1+2\nu_m)\rho_m b_{3c}\right] \equiv e_5^{A_n}$$

$$\sigma_{33}^{A_n(1)} = \sigma_{33}^{A_n(2)} \Rightarrow$$
$$\eta_a^{A_n(m)}(-1)^{m-1}2a_{1a}^{(m)} + \eta_b^{A_n(m)} iQ_b\left[2-\sqrt{\rho_m} + (3-4\nu_m^2)\sqrt{\nu_m}\right]$$
$$+\eta_c^{A_n(m)}2i\left[\rho_m b_{2c} + (-1)^{m-1}a_{2c}^{(m)} + \sqrt{\theta}\left((-1)^{m-1}a_{3c}^{(m)} - \rho_m b_{3c}\left(1+2\nu_m(1-\theta)\right)\right)\right]$$
$$+\eta_d^{A_n(m)}2i\left[(-1)^{m-1}a_{2d}^{(m)} - \overline{b}^{(m)}\right] \equiv e_6^{A_n};$$

$$\sigma_{12}^{A_n(1)} = \sigma_{12}^{A_n(2)} \Rightarrow$$
$$-C_m + \eta_a^{A_n(m)}2a_{1a}^{(m)} + \eta_b^{A_n(m)}(-1)^{m-1}2iQ_b$$
$$+\eta_c^{A_n(m)}2i\sum_{s=1}^{3}\left[a_{sc}^{(m)} + (-1)^m 2\nu_m\rho_m(-1)^{s-1}b_{sc}\right]$$
$$+\eta_d^{A_n(m)}2i\left[a_{1d}^{(m)} + a_{2d}^{(m)} + (-1)^m 4\nu_m\overline{b}^{(m)}\right] \equiv e_{71}^{A_n},$$

$$-\nu_m C_m + \eta_a^{A_n(m)}2a_{1a}^{(m)} + \eta_b^{A_n(m)}(-1)^{m-1}2iQ_b$$
$$+\eta_c^{A_n(m)}2i\left[a_{2c}^{(m)} + (-1)^{m-1}2\nu_m\rho_m b_{2c}\right] + \eta_d^{A_n(m)}2i\left[a_{2d}^{(m)} + (-1)^m 2\nu_m\overline{b}^{(m)}\right] \equiv e_{72}^{A_n};$$

$$\sigma_{13}^{A_n(1)} = \sigma_{13}^{A_n(2)} \Rightarrow$$
$$\eta_a^{A_n(m)}(-1)^{m-1}2a_{1a}^{(m)} + \eta_b^{A_n(m)} iQ_b\left[2+\nu_m(3-4\nu_m)-\rho_m\right]$$
$$+\eta_c^{A_n(m)}2i\left[\rho_m b_{2c} + (-1)^{m-1}a_{2c}^{(m)} + \theta\left((-1)^{m-1}a_{3c}^{(m)} - \rho_m b_{3c}\right)\right]$$
$$+\eta_d^{A_n(m)}2i\left[(-1)^{m-1}a_{2d}^{(m)} - \overline{b}^{(m)}\right] \equiv e_8^{A_n};$$

$$\sigma_{23}^{A_n(1)} = \sigma_{23}^{A_n(2)} \Rightarrow$$
$$\text{expression } \sigma_{72}^{A_n} \text{ above;} \tag{43}$$

where all $e_i^{A_n}(m)$ are constant with m=1 and 2 (*i.e.* $e_i^{A_n}(1) = e_i^{A_n}(2)$). The equations (43) have eight unknowns $\eta_i^{A_n(m)}$ (i= a to d; m=1 and 2).

The methodology of the solution may be: (*i*) express the $\eta_i^{A_n(2)}$ as a function of the $\eta_i^{A_n(1)}$ giving four relations, (*ii*) report these relations in four independent equations in (43); we have then a linear system of four equations with unknowns $\eta_i^{A_n(1)}$ that can be resolved by the usual classical method with determinants. The result is four expressions linking the $\eta_i^{A_n(1)}$ with the elastic constants of the mediums *m*=1 and 2. (*iii*) Come back to the $\eta_i^{A_n(2)}$ relations to find their respective values as a function of the elastic constants. The pair $(i\mu_2 e_1^{A_n}(1) - \mu_2 e_2^{A_n}(1);\, e_5^{A_n}(1))$ is a linear system of two equations with unknowns $\eta_b^{A_n(2)}$ and $\eta_c^{A_n(2)}$ that provides relations of the formers with respect to the $\eta_i^{A_n(1)}$. The result is that the relations for both $\eta_b^{A_n(2)}$ and $\eta_c^{A_n(2)}$ only depend on $\eta_b^{A_n(1)}$ and $\eta_c^{A_n(1)}$. These are introduced in the pair $(\mu_2 e_1^{A_n}(1);\, e_3^{A_n}(1))$ that becomes a linear system of two equations with two unknowns $\eta_a^{A_n(2)}$ and $\eta_d^{A_n(2)}$, the resolution of which, provide the relations for these parameters. It is found that both parameters depend on all the $\eta_i^{A_n(1)}$ (*i*= *a* to *d*). Hence, we can write

$$\begin{pmatrix} \eta_a^{A_n(2)} \\ \eta_b^{A_n(2)} \\ \eta_c^{A_n(2)} \\ \eta_d^{A_n(2)} \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ 0 \\ D \end{pmatrix} + \begin{pmatrix} A_a & A_b & A_c & A_d \\ 0 & B_b & B_c & 0 \\ 0 & C_b & C_c & 0 \\ D_a & D_b & D_c & D_d \end{pmatrix} \begin{pmatrix} \eta_a^{A_n(1)} \\ \eta_b^{A_n(1)} \\ \eta_c^{A_n(1)} \\ \eta_d^{A_n(1)} \end{pmatrix} \tag{44}$$

where *A* and *D* and the elements of the 4×4 matrix depend on the elastic constants of both media; these are listed in the Appendix. Next, the following notations are used:

$$e_i^{A_n}(m) = t_i^{(m)} + t_{ia}^{(m)}\eta_a^{A_n(m)} + t_{ib}^{(m)}\eta_b^{A_n(m)} + t_{ic}^{(m)}\eta_c^{A_n(m)} + t_{id}^{(m)}\eta_d^{A_n(m)}. \tag{45}$$

The comparison of (45) with (43) gives without ambiguity the values of the coefficients $t_i^{(m)}$ and $t_{ij}^{(m)}$ required by the resolution methodology. The general $e_i^{A_n}(2) = e_i^{A_n}(1)$ becomes

$$\begin{aligned} &\eta_a^{A_n(1)}\left(t_{ia}^{(2)}A_a + t_{id}^{(2)}D_a - t_{ia}^{(1)}\right) + \eta_b^{A_n(1)}\left(t_{ia}^{(2)}A_b + t_{ib}^{(2)}B_b + t_{ic}^{(2)}C_b + t_{id}^{(2)}D_b - t_{ib}^{(1)}\right) \\ &+ \eta_c^{A_n(1)}\left(t_{ia}^{(2)}A_c + t_{ib}^{(2)}B_c + t_{ic}^{(2)}C_c + t_{id}^{(2)}D_c - t_{ic}^{(1)}\right) + \eta_d^{A_n(1)}\left(t_{ia}^{(2)}A_d + t_{id}^{(2)}D_d - t_{id}^{(1)}\right) \\ &= t_i^{(1)} - \left(t_i^{(2)} + t_{ia}^{(2)}A + t_{id}^{(2)}D\right). \end{aligned} \tag{46}$$

For i = 1 to 4, we arrive at a linear nonhomogenous system of four equations with unknowns $\eta_i^{A_n(1)}$ (i= a to d), to be resolved to provide values to $\eta_i^{A_n(1)}$. The system has the form

$$\begin{cases} a_{11}\eta_a^{A_n(1)} + a_{12}\eta_b^{A_n(1)} + a_{13}\eta_c^{A_n(1)} + a_{14}\eta_d^{A_n(1)} = b_1 \\ a_{21}\eta_a^{A_n(1)} + a_{22}\eta_b^{A_n(1)} + a_{23}\eta_c^{A_n(1)} + a_{24}\eta_d^{A_n(1)} = b_2 \\ a_{31}\eta_a^{A_n(1)} + a_{32}\eta_b^{A_n(1)} + a_{33}\eta_c^{A_n(1)} + a_{34}\eta_d^{A_n(1)} = b_3 \\ a_{41}\eta_a^{A_n(1)} + a_{42}\eta_b^{A_n(1)} + a_{43}\eta_c^{A_n(1)} + a_{44}\eta_d^{A_n(1)} = b_4 \end{cases} . \tag{47}$$

In matrix notation, (47) is associated with a 4×4 matrix (A) with elements a_{ij} (i,j= 1 to 4) and (B) a 4×1 matrix with elements b_i (i=1 to 4). Thus,

$$\begin{pmatrix} \eta_a^{A_n(1)} \\ \eta_b^{A_n(1)} \\ \eta_c^{A_n(1)} \\ \eta_d^{A_n(1)} \end{pmatrix} = (A^{-1}) \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} \tag{48}$$

where (A^{-1}) is the inverse matrix of (A), with elements

$$(A^{-1})_{ij} = \frac{1}{\det(A)}(-1)^{i+j}\Delta_{ji} \; ; \tag{49}$$

$\det(A)$ is a 4th-order determinant associated with matrix (A) and Δ_{ji}, a 3rd-order determinant, associated with a 3×3 matrix obtained from (A) by removing the row (j) and column (i) of element a_{ji}. With the help of relation $e_i^{A_n}(2) = e_i^{A_n}(1)$ (46), the following identification holds:

$$\begin{aligned} b_i &= t_i^{(1)} - \left(t_i^{(2)} + t_{ia}^{(2)}A + t_{id}^{(2)}D\right) \\ a_{i1} &= t_{ia}^{(2)}A_a + t_{id}^{(2)}D_a - t_{ia}^{(1)} \\ a_{i2} &= t_{ia}^{(2)}A_b + t_{ib}^{(2)}B_b + t_{ic}^{(2)}C_b + t_{id}^{(2)}D_b - t_{ib}^{(1)} \\ a_{i3} &= t_{ia}^{(2)}A_c + t_{ib}^{(2)}B_c + t_{ic}^{(2)}C_c + t_{id}^{(2)}D_c - t_{ic}^{(1)} \\ a_{i4} &= t_{ia}^{(2)}A_d + t_{id}^{(2)}D_d - t_{id}^{(1)} . \end{aligned} \tag{50}$$

We carefully choose the following four equations (Ei), i=1 to 4:

$$
\begin{aligned}
&(E1): \quad e_{71}^{A_n}(2) - e_{72}^{A_n}(2) = e_{71}^{A_n}(1) - e_{72}^{A_n}(1) \\
&(E2): \quad e_{71}^{A_n}(2) = e_{71}^{A_n}(1) \\
&(E3): \quad e_{5}^{A_n}(2) = e_{5}^{A_n}(1) \\
&(E4): \quad e_{1}^{A_n}(2) = e_{1}^{A_n}(1)\,; \qquad (51)
\end{aligned}
$$

these lead to following identifications:

$$
\begin{aligned}
b_1 &= C_2(1-\nu_2) - C_1(1-\nu_1) - 2i\left(a_{1d}^{(2)} + 2\nu_2\overline{b}^{(2)}\right)D\,, \\
a_{11} &= 2i\left(a_{1d}^{(2)} + 2\nu_2\overline{b}^{(2)}\right)D_a\,, \\
a_{12} &= 2i\left(a_{1c}^{(2)} + a_{3c}^{(2)} + 2\nu_1\nu_2(b_{1c}+b_{3c})\right)C_b + 2i\left(a_{1d}^{(2)} + 2\nu_2\overline{b}^{(2)}\right)D_b\,, \\
a_{13} &= 2i\left(a_{1c}^{(2)} + a_{3c}^{(2)} + 2\nu_1\nu_2(b_{1c}+b_{3c})\right)C_c + 2i\left(a_{1d}^{(2)} + 2\nu_2\overline{b}^{(2)}\right)D_c \\
&\qquad - 2i\left(a_{1c}^{(1)} + a_{3c}^{(1)} - 2\nu_1\nu_2(b_{1c}+b_{3c})\right), \\
a_{14} &= 2i\left(a_{1d}^{(2)} + 2\nu_2\overline{b}^{(2)}\right)D_d - 2i\left(a_{1d}^{(1)} - 2\nu_1\overline{b}^{(1)}\right);
\end{aligned}
$$

$$
\begin{aligned}
b_2 &= C_2 - C_1 - 2a_{1a}^{(2)}A - 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\overline{b}^{(2)}\right)D\,, \\
a_{21} &= 2a_{1a}^{(2)}A_a + 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\overline{b}^{(2)}\right)D_a - 2a_{1a}^{(1)}\,, \\
a_{22} &= 2a_{1a}^{(2)}A_b - 2iQ_bB_b + 2i\sum_{s=1}^{3}\left[a_{sc}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}\right]C_b \\
&\qquad + 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\overline{b}^{(2)}\right)D_b - 2iQ_b\,, \\
a_{23} &= 2a_{1a}^{(2)}A_c - 2iQ_bB_c + 2i\sum_{s=1}^{3}\left[a_{sc}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}\right]C_c \\
&\qquad + 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\overline{b}^{(2)}\right)D_c - 2i\sum_{s=1}^{3}\left[a_{sc}^{(1)} + 2\nu_1\nu_2(-1)^{s}b_{sc}\right], \\
a_{24} &= 2a_{1a}^{(2)}A_d + 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\overline{b}^{(2)}\right)D_d - 2i\left(a_{1d}^{(1)} + a_{2d}^{(1)} - 4\nu_1\overline{b}^{(1)}\right);
\end{aligned}
$$

$$
\begin{aligned}
b_3 &= 0\,, \\
a_{31} &= 0\,, \\
a_{32} &= Q_b\left((1-\nu_2)\sqrt{\nu_2} + (1-\nu_1)\sqrt{\nu_1}\right)B_b \\
&\qquad + 2(1-\theta)\sqrt{\theta}\left(a_{3c}^{(2)} - (1-2\nu_2)\nu_1 b_{3c}\right)C_b - Q_b\left((1-\nu_2)\sqrt{\nu_2} + (1-\nu_1)\sqrt{\nu_1}\right), \\
a_{33} &= Q_b\left((1-\nu_2)\sqrt{\nu_2} + (1-\nu_1)\sqrt{\nu_1}\right)B_c \\
&+ 2(1-\theta)\sqrt{\theta}\left(a_{3c}^{(2)} - (1-2\nu_2)\nu_1 b_{3c}\right)C_c + 2(1-\theta)\sqrt{\theta}\left(a_{3c}^{(1)} + (1-2\nu_1)\nu_2 b_{3c}\right), \\
a_{34} &= 0\,;
\end{aligned}
$$

$$b_4 = \frac{1}{\mu_2}\left\{2a_{1a}^{(2)}A + 2i\left(a_{2d}^{(2)} + \overline{b}^{(2)}\right)D\right\},$$

$$a_{41} = -\frac{1}{\mu_2}\left\{2a_{1a}^{(2)}A_a + 2i\left(a_{2d}^{(2)} + \overline{b}^{(2)}\right)D_a\right\} - 2\frac{a_{1a}^{(1)}}{\mu_1},$$

$$a_{42} = \frac{1}{\mu_2}\left\{-2a_{1a}^{(2)}A_b + iQ_b\left[2 + \nu_2(3-4\nu_2) - \nu_1\right]B_b + 2i\left[\nu_1 b_{2c} - a_{2c}^{(2)} - \theta\left(a_{3c}^{(2)} + \nu_1 b_{3c}\right)\right]C_b - 2i\left(a_{2d}^{(2)} + \overline{b}^{(2)}\right)D_b\right\} - \frac{iQ_b}{\mu_1}\left(2 + \nu_1(3-4\nu_1) - \nu_2\right),$$

$$a_{43} = \frac{1}{\mu_2}\left\{-2a_{1a}^{(2)}A_c + iQ_b\left(2 + \nu_2(3-4\nu_2) - \nu_1\right)B_c + 2i\left(\nu_1 b_{2c} - a_{2c}^{(2)} - \theta\left(a_{3c}^{(2)} + \nu_1 b_{3c}\right)\right)C_c - 2i\left(a_{2d}^{(2)} + \overline{b}^{(2)}\right)D_c\right\} - \frac{2i}{\mu_1}\left(\nu_2 b_{2c} + a_{2c}^{(1)} + \theta\left(a_{3c}^{(1)} - \nu_2 b_{3c}\right)\right),$$

$$a_{44} = -\frac{1}{\mu_2}\left\{2a_{1a}^{(2)}A_d + 2i\left(a_{2d}^{(2)} + \overline{b}^{(2)}\right)D_d\right\} - \frac{2i}{\mu_1}\left(a_{2d}^{(1)} - \overline{b}^{(1)}\right). \quad (52)$$

(52) gives the matrices (A) and (B) and (48) the corresponding solutions $\eta_i^{A_n(1)}$ ($i= a$ to d). It is then possible to reach the associated $\eta_j^{A_n(2)}$ ($j= a$ to d) with the help of (44).

The solution $\eta_i^{A_n(m)}$ ($i= a$ to d; m=1 and 2) must satisfies the entire (43). Next, we shall restrict ourselves to demonstrate that we have effectively $e_5^{A_n}(1) = e_5^{A_n}(2)$ that only involves two parameters $\eta_i^{A_n(m)}$ ($i= b$ and c). We find

$$\eta_b^{A_n(1)} = \frac{a_{33}}{\det(A)}\left\{b_1(a_{41}a_{24} - a_{21}a_{44}) + b_2(a_{11}a_{44} - a_{41}a_{14}) + b_4(a_{21}a_{14} - a_{11}a_{24})\right\}$$

$$\eta_c^{A_n(1)} = -\frac{a_{32}}{a_{33}}\eta_b^{A_n(1)}. \quad (53)$$

Using (44), we get

$$\eta_b^{A_n(2)} = \frac{1}{a_{33}}\left(a_{33}B_b - a_{32}B_c\right)\eta_b^{A_n(1)}$$

$$\eta_c^{A_n(2)} = \frac{1}{a_{33}}\left(a_{33}C_b - a_{32}C_c\right)\eta_b^{A_n(1)} \quad (54)$$

so that

$$\eta_c^{A_n(2)} = \frac{a_{33} C_b - a_{32} C_c}{a_{33} B_b - a_{32} B_c} \eta_b^{A_n(2)} . \tag{55}$$

It is then easy to show that

$$e_5^{A_n}(1) = \Big(Q_b \big[(1-\nu_1)\sqrt{\nu_1} + (1-\nu_2)\sqrt{\nu_2} \big] + \frac{2(1-\theta)\sqrt{\theta}\, a_{32}}{a_{33}} \big[a_{3c}^{(1)} + (1-2\nu_1)\nu_2 b_{3c} \big] \Big) \eta_b^{A_n(1)} = e_5^{A_n}(2) \cdot \tag{56}$$

(56) is in favour of the rightness of our calculations. The associated fields $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ (41) must be continuous at the crossing of the interface, at $P_S = (x_1, x_2 = \xi, x_3)$, to first order in ξ_n. The continuity of $\sigma_{23}^{A_n(m)}$ on the interface permits to write the associated singularity term (to first order in ξ_n) as

$$\sigma_{23}^{A_n(m)}(x_1, 0, x_3) \equiv -\frac{\partial A_n}{\partial x_3} \frac{e_{72}^{A_n}}{x_1} . \tag{57}$$

(57) is useful because it contributes a non-zero value to the crack extension force when a non-planar interface crack with a wavy front is loaded in tension (see [3, 5, 7] for non-planar wavy crack propagation in a homogeneous medium, the analogous case). In summary, the elastic fields ($\vec{u}^{(m)}$ and $(\sigma)^{(m)}$: (40)) due to an interfacial sinusoidal edge dislocation ***(Figure 1)*** to first order in the perturbation, is the sum of two terms: the first one corresponds to the elastic fields ($\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$: (20)) produced by an interface straight edge dislocation, lying on a planar interface, whose expressions have been provided in Section 3.1, under continuity requirement at the crossing of the interface; the second term is an oscillating expression ($\vec{u}^{A_n(m)}, (\sigma)^{A_n(m)}$: (41)), proportional to the perturbation $A_n(x_3) = \xi_n \sin \kappa_n x_3$ or its partial derivative $\partial A_n / \partial x_3$. This latter expression can be written as a linear combination (42) of partial elastic fields $\vec{u}_i^{A_n(m)V}$ (*i*= *a* to *d*) ((36) to (39)). The associated proportionality coefficients $\eta_i^{A_n(m)}$ ((44), (48)) have determined under the assumption that the elastic fields ($\vec{u}^{(m)}, (\sigma)^{(m)}$) must be continuous on crossing the sinusoidal interface.

IV - DISCUSSION

We would like first to mention that the values of the elastic fields ($\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$: (20)) on the interface points $(x_1, x_2 = 0, x_3)$ are independent of $\overline{V_c}$ (see (30) to (33)). This suggests that $\overline{V_c}$ could be set equal to zero with no change in the elastic fields; the associated couple (see (16)) $(\overline{\alpha}_{2c} \equiv 0, \overline{\beta}_{2c}^{(m)})$ corresponds to calculating $\overline{\beta}_2^{(m)}$ using (14 *i* and *j*) and imposing $\overline{\alpha}_2^{(m)} \equiv 0$. Expressions for the displacement and stress fields of interface straight edge dislocations with their associated Airy stress functions have been given [13, 14]. In the geometry of the ***Figure*** **1**, the calculated shear stress $\sigma_{12}(x_1, x_2 = 0, x_3)$ is zero on the interface for an edge dislocation with Burgers vector $\vec{b} = (0, b, 0)$ perpendicular to the interface. Later on, [15] has stressed that this shear stress contains a term proportional to the Dirac delta function; this result has been incorporated in a number of analyses of the propagation of the interface crack under load ([16-19], among others). Incorporating a Dirac delta function in the value of the shear stress on the interface is a clear indication that the elasticity solutions given by Dundurs and Mura [13] are partial. In the present study, a Dirac delta function is present in the complete elastic fields (Section 3.1.4) and pertains to the partial elastic fields $\vec{u}_c^{(0)(m)V}$ $(\sigma)_c^{(0)(m)V}$ only (see (19), Section 3.1.1). This suggests that the results of the present study are potentially more general; however, discrepancies between our expressions for $\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3)$ (30) and $\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3)$ (32) and those given below by [17] are observed. In the geometry of the ***Figure*** **1**, their results are (solid *S*1):

$$\sigma_{12}^{(1)}(x_1, x_2 = 0, x_3) = -\frac{2b\mu_1\mu_2[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]}{[\mu_1 + \mu_2(3-4\nu_1)][\mu_2 + \mu_1(3-4\nu_2)]}\delta(x_1)\,;$$

$$\sigma_{22}^{(1)}(x_1, x_2 = 0, x_3) = \frac{4b\mu_1\mu_2[\mu_1(1-2\nu_2) + \mu_2(1-2\nu_1)]}{\pi[\mu_1 + \mu_2(3-4\nu_1)][\mu_2 + \mu_1(3-4\nu_2)]}\frac{1}{x_1}\,.$$

Their shear stress $\sigma_{12}^{(1)}$ changes sign on inverting the elastic constants indicating that it is discontinuous on crossing the interface from solid *S*1 ($\sigma_{12}^{(1)}$) to solid *S*2 ($\sigma_{12}^{(2)}$). In contrast our result for $\sigma_{12}^{(0)(m)}$ (30) is continuous at the crossing of the interface. This indicates an essential difference between the interface boundary conditions in both studies. Their $\sigma_{22}^{(1)}$ above is continuous across the interface like our value (32).

V - CONCLUSION

In the present study, Galerkin vectors have been used (Section 2.2) and associated interface boundary conditions have been displayed. These conditions can be decomposed into two groups: (1) the first group corresponds to a planar interface with a straight edge dislocation at the origin (13); (2) the second group corresponds to terms proportional to the sinusoid or its spatial derivative with respect to x_3 in the elastic fields expressed to first order with respect to the perturbation $A_n(x_3) = \xi_n \sin \kappa_n x_3$ (14). Satisfying both boundary conditions leads to terms of first order with respect to ξ_n in the elastic fields of a sinusoidal edge dislocation (see ***Figure 1***). The terms of zero order ($\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$) correspond to the elastic fields of an interface straight edge dislocation. These results contain the Dirac delta function in the shear stresses on the interface. We have compared these findings with those previously published on similar problems. The terms of first order are proportional to A_n or its partial derivative $\partial A_n / \partial x_3$. Among the additional oscillating stresses, $\sigma_{23}^{(m)}$ is the only one that possesses a singularity of the $1/x_1$ type on the interface. This stress contributes to the expression of the crack extension force when a non-planar interface crack with a wavy front is loaded in tension.

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APPENDIX

The relation (44) expresses the $\eta_i^{A_n(2)}$ (i=a to d) as linear combinations of the $\eta_i^{A_n(1)}$. The associated proportionality coefficients have been determined by a procedure expounded in Section 3.2.3. We obtain

$$A = \frac{\mu_2^2 t_{1d}^{(2)}\left(t_3^{(2)} - t_3^{(1)}\right)}{\Delta_{34}^{(2)}}, \quad A_a = \frac{\mu_2^2\left(t_{3d}^{(2)}t_{1a}^{(1)} - t_{1d}^{(2)}t_{3a}^{(1)}\right)}{\Delta_{34}^{(2)}}, \; A_d = \frac{\mu_2^2\left(t_{3d}^{(2)}t_{1d}^{(1)} - t_{1d}^{(2)}t_{3d}^{(1)}\right)}{\Delta_{34}^{(2)}},$$

$$A_b = \frac{\mu_2^2}{\mu_1\Delta_{12}^{(2)}\Delta_{34}^{(2)}}\left\{\mu_1\Delta_{12}^{(2)}\left(t_{3d}^{(2)}t_{1b}^{(1)} - t_{1d}^{(2)}t_{3b}^{(1)}\right) + [\mu_1 R_{12}^{(2)}R_{21}^{(1)} - \mu_2 R_{22}^{(2)}R_{11}^{(1)}]\right.$$
$$\left.\times\,(t_{3d}^{(2)}t_{1b}^{(2)} - t_{1d}^{(2)}t_{3b}^{(2)}) + [\mu_2 R_{21}^{(2)}R_{11}^{(1)} - \mu_1 R_{11}^{(2)}R_{21}^{(1)}](t_{3d}^{(2)}t_{1c}^{(2)} - t_{1d}^{(2)}t_{3c}^{(2)})\right\},$$

$$A_c = \frac{\mu_2^2}{\mu_1\Delta_{12}^{(2)}\Delta_{34}^{(2)}}\left\{\mu_1\Delta_{12}^{(2)}\left(t_{3d}^{(2)}t_{1c}^{(1)} - t_{1d}^{(2)}t_{3c}^{(1)}\right) + [\mu_1 R_{12}^{(2)}R_{22}^{(1)} - \mu_2 R_{22}^{(2)}R_{12}^{(1)}]\right.$$
$$\left.\times\,(t_{3d}^{(2)}t_{1b}^{(2)} - t_{1d}^{(2)}t_{3b}^{(2)}) + [\mu_2 R_{21}^{(2)}R_{12}^{(1)} - \mu_1 R_{11}^{(2)}R_{22}^{(1)}](t_{3d}^{(2)}t_{1c}^{(2)} - t_{1d}^{(2)}t_{3c}^{(2)})\right\};$$

$$B_b = \frac{\mu_2 R_{22}^{(2)}R_{11}^{(1)} - \mu_1 R_{12}^{(2)}R_{21}^{(1)}}{\mu_1\Delta_{12}^{(2)}}, \quad B_c = \frac{\mu_2 R_{22}^{(2)}R_{12}^{(1)} - \mu_1 R_{12}^{(2)}R_{22}^{(1)}}{\mu_1\Delta_{12}^{(2)}};$$

$$C_b = \frac{\mu_1 R_{11}^{(2)}R_{21}^{(1)} - \mu_2 R_{21}^{(2)}R_{11}^{(1)}}{\mu_1\Delta_{12}^{(2)}}, \quad C_c = \frac{\mu_1 R_{11}^{(2)}R_{22}^{(1)} - \mu_2 R_{21}^{(2)}R_{12}^{(1)}}{\mu_1\Delta_{12}^{(2)}};$$

$$D = \frac{\mu_2^2 t_{1a}^{(2)}\left(t_3^{(1)} - t_3^{(2)}\right)}{\Delta_{34}^{(2)}}, \quad D_a = \frac{\mu_2^2\left(t_{1a}^{(2)}t_{3a}^{(1)} - t_{3a}^{(2)}t_{1a}^{(1)}\right)}{\Delta_{34}^{(2)}}, \quad D_d = \frac{\mu_2^2\left(t_{1a}^{(2)}t_{3d}^{(1)} - t_{3a}^{(2)}t_{1d}^{(1)}\right)}{\Delta_{34}^{(2)}},$$

$$D_b = -\frac{\mu_2^2}{\mu_1 \Delta_{12}^{(2)} \Delta_{34}^{(2)}} \left\{ \mu_1 \Delta_{12}^{(2)} \left(t_{3a}^{(2)} t_{1b}^{(1)} - t_{1a}^{(2)} t_{3b}^{(1)} \right) + [\mu_1 R_{12}^{(2)} R_{21}^{(1)} - \mu_2 R_{22}^{(2)} R_{11}^{(1)}] \right.$$
$$\left. \times (t_{3a}^{(2)} t_{1b}^{(2)} - t_{1a}^{(2)} t_{3b}^{(2)}) + [\mu_2 R_{21}^{(2)} R_{11}^{(1)} - \mu_1 R_{11}^{(2)} R_{21}^{(1)}](t_{3a}^{(2)} t_{1c}^{(2)} - t_{1a}^{(2)} t_{3c}^{(2)}) \right\},$$

$$D_c = -\frac{\mu_2^2}{\mu_1 \Delta_{12}^{(2)} \Delta_{34}^{(2)}} \left\{ \mu_1 \Delta_{12}^{(2)} \left(t_{3a}^{(2)} t_{1c}^{(1)} - t_{1a}^{(2)} t_{3c}^{(1)} \right) + [\mu_1 R_{12}^{(2)} R_{22}^{(1)} - \mu_2 R_{22}^{(2)} R_{12}^{(1)}] \right.$$
$$\left. \times (t_{3a}^{(2)} t_{1b}^{(2)} - t_{1a}^{(2)} t_{3b}^{(2)}) + [\mu_2 R_{21}^{(2)} R_{12}^{(1)} - \mu_1 R_{11}^{(2)} R_{22}^{(1)}](t_{3a}^{(2)} t_{1c}^{(2)} - t_{1a}^{(2)} t_{3c}^{(2)}) \right\};$$

where

$$\Delta_{12}^{(2)} = R_{11}^{(2)} R_{22}^{(2)} - R_{21}^{(2)} R_{12}^{(2)},$$
$$\Delta_{34}^{(2)} = \mu_2^2 \left(t_{1a}^{(2)} t_{3d}^{(2)} - t_{3a}^{(2)} t_{1d}^{(2)} \right);$$

$$R_{11}^{(m)} = Q_b \left[\rho_m - \sqrt{\rho_m} - (3 - 4\nu_m)(\nu_m - \sqrt{\nu_m}) \right],$$
$$R_{12}^{(m)} = 2(\theta - \sqrt{\theta}) \left[(-1)^m a_{3c}^{(m)} + \rho_m b_{3c} \right],$$
$$R_{22}^{(m)} = 2(1 - \theta)\sqrt{\theta} \left[(-1)^m a_{3c}^{(m)} - (1 - 2\nu_m)\rho_m b_{3c} \right],$$
$$R_{21}^{(m)} = Q_b \left[(1 - \nu_m)\sqrt{\nu_m} + (1 - \rho_m)\sqrt{\rho_m} \right], \quad m = 1 \text{ and } 2.$$

We recall, for $t_{ij}^{(m)}$ and $t_i^{(m)}$, to see (45); all the various other parameters are defined previously in the text.